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Declaration of authorship

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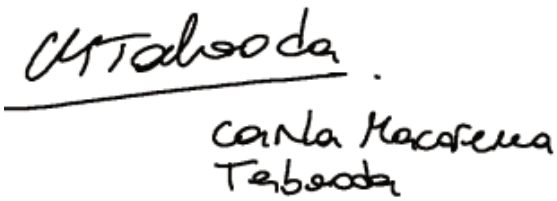

Abstract

Since economies of different countries and their markets do not move in the same direction, the opportunity to invest in different regions can reduce risk and/or increase returns. Technological changes, as well as globalisation and financial deregulation, have led to significant changes in the international mobility of capital, with both positive and negative effects, which have modified the functioning of financial markets and the development of interactions between countries.

International portfolio diversification has become a key strategy to mitigate risk and optimise returns, as geographical optimisation by allocating assets across countries allows investors to take advantage of the benefits of each market while reducing their exposure to specific shocks of a country or region.

This analysis aims to determine whether international diversification is better in terms of return and risk than diversification across a large portfolio of assets but from a single country or region.

In addition, the secondary objectives that this research follows are:

- Describe the asset allocation theory as well as its international outlook.
- Apply the asset allocation theory to the selected indices.
- Find different internationally diversified portfolios according to different risk profiles.

This will be done through the analysis of the following market indices: S&P 500, Euro Stoxx 50, Nikkei 225, FTSE 100, Bovespa and S&P/BMV IPC (MXX) for the period between January 2012 to December 2022.

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List of Abbreviations

BMV: Mexican Stock Exchange

Bovespa: Bolsa de Valores do Estado de São Paulo

CAL: Capital Allocation Line

CAPM: Capital Asset Pricing Model

CMP: Capital Market Line

DRs: depositary receipts

ETF's: exchange traded funds

FTSE: Financial Times Stock Exchange

FX: Foreign Exchange Market

GICS: Global Industry Classification Standard

IAPM: International Asset Pricing Model

ICT: Information and communication technology

IN: Tradability Index

NASDAQ: National Association of Security Dealers Automated Quotations

NYSE: New York Stock Exchange

OECD: Organisation for Economic Co-operation and Development

SML: Security Marlet Line

S&P 500: Standard & Poor's 500

TMI: Total Market Index

UK: United Kingdom

US: United States of America

Introduction

The main objective in portfolio management is to achieve the highest return with the available assets, which leads to the challenge of determining in which assets to invest in order to maximise returns.

Financial markets have changed over the years and the main reasons are globalization, improvement of technology and communication systems. Other contributing factors are financial deregulation and the free capital mobility between countries, which has opened up an unlimited universe of investment options.

Information and communication technology (ICT) sector grew by an average of 6.3% between 2013 and 2023, that means three times faster than the total economy of OECD countries. The leading countries were United Kingdom, Belgium, Germany, Austria and Netherlands (OECD, 2024). This is evidenced by the increasing presence of FinTech's offering different types of investment options, the growing number of digital services offered by banks, and the increasing use of artificial intelligence tools.

In this scenario, the possibilities of investing in different countries are expanding even in less developed countries, where there is also wide access to these technologies.

This is why international portfolio diversification has become a key strategy to mitigate risk and optimise returns, as geographical optimisation by allocating assets across countries allows investors to take advantage of the benefits of each market while reducing their exposure to specific shocks of a country or region.

However, it should be mentioned, that increased integration can also generate new economic problems, as risks are transmitted between economies, for example, in the event of crises (Balcilar, et al., 2019). Capital flows can increase economic fluctuations since countries are exposed to events related to modifications in economic context outside the country (Stiglitz, 2000). In this context, new financial products have been developed, to take advantage of the changes and hedge against the risks that this new environment has created.

Taking into account the above-mentioned changes and considering today's access to foreign markets, this research will conduct an analysis of internationally diversified equity portfolios. Using the Markowitz theory and analysing different stock markets from five different regions around the world, for a period of eleven years, from January 2012 to December 2022, a risk-return comparison will be made between a domestic and an internationally diversified portfolio.

The indices that will be considered are: S&P 500 (United States), Euro Stoxx 50 (Europe), Nikkei 225 (Japan), FTSE 100 (United Kingdom), Bovespa (Brazil) and S&P/BMV IPC (Mexico), this will make it possible to cover different realities and contexts, both in terms of economic and market development as well as integration into the world.

In addition, an optimal international portfolio for different risk profiles will be determined. Through this analysis, it will be possible on the one hand to check whether or not the theory holds true for the selected countries and period, and on the other hand, to show how it is appropriate to combine equity assets when investing internationally.

1. Research Design

The structure will be divided into different sections, being the first one an explanation of the problem statement and objectives as well as the methodology that will be used and how the analysis will be carried out.

The second section will provide a literature review of previous research on the same topic, which will highlight previous findings on international investment, both theoretically and in practice. The exploration of similar studies, for other time periods and/or regions, will serve as a basis for further analysis of the topic and the state of art.

The third part will consist in a first term of a brief description of changes and developments that have taken place on the financial system as well as their effects, and then it will provide an overview of the portfolio theory and optimisation literature, and also of the International Capital Asset Pricing Model.

It will also include a description of different investment alternatives abroad, as well as a summary of the benefits and difficulties involved. Finally, the selected market indices from different regions will be presented, together with a brief explanation of the macroeconomic context of each country.

The fourth section will present and analyse the collected data from the different market indices. This is the section where the theory will be applied to the concrete case with the aim to solve the research question.

To finalize, the fifth section will present the results as well as the corresponding recommendations.

1.1 Problem statement

The Markowitz model (1952) is a well-known tool for creating investment portfolios, which makes it possible to find for each investor a selection of assets in which for a given level of risk the returns can be maximised. This can be achieved through the selection of assets without perfect correlation.

Since economies of different countries and their markets do not move in the same direction, the opportunity to invest in different regions can reduce risk and/or increase returns. As stated in portfolio theory, diversification is a primary objective in any investment decision, therefore, this thesis will investigate how equity allocation across countries can be used to optimize portfolios.

In this context, the following research question arises: *Is it possible to increase the efficiency of an investment equity portfolio through international diversification?*

1.2 Objectives

The main objective is to analyse the performance of indices from different regions over time in isolation and then make comparisons with an optimal international portfolio, in order to analyse whether international diversification would have been a more efficient investment option.

This analysis would be useful to determine whether international diversification is better in terms of return and risk than diversification across a large portfolio of assets but from a single country or region.

Secondary objectives that this research follows are:

- Describe the asset allocation theory as well as its international outlook.
- Apply the asset allocation theory to the selected indices.
- Find different internationally diversified portfolios according to different risk profiles.

1.3 Methodology

1.3.1 Type of design

This research will have a mixed approach, in a first instance a qualitative approach will be used, which is necessary to know the existing studies on this

topic, as well as to deeply understand the importance of diversification, through a review of the literature. In addition, a description of the stock market indices that will be used later on will be made, also briefly inquiring about the context of each country.

Subsequently, a quantitative approach will be used to analyse the selected indices, the correlation between them and to make comparisons.

In terms of scope, it will be a descriptive-correlational analysis, since it will be necessary to identify and clarify theoretical concepts first. Once the conceptual assumptions have been established, a more detailed analysis will be carried out through the examination of the collected market data.

Finally, once analysed the information using the theoretical tools and making comparisons between indices, the main objective will be tested.

The results will also provide background for future portfolio construction forecasts, allowing suggestions on how to combine international equities in order to achieve an efficient portfolio.

1.3.2 Data collection

Information will be retrieved from different sources to create the theoretical background and also to get to know the state of the art, such as research papers, articles, journals, books, etcetera.

For the quantitative analysis the data will be collected from Investing.com for the following indices, representative of different regions around the world: S&P 500, Euro Stoxx 50, Nikkei 225, FTSE 100, Bovespa and S&P/BMV IPC (MXX) for the period between January 2012 to December 2022; all of them will be expressed in USD as the common currency, also using Investing.com rates exchange for the same period.

1.3.3 Data analysis

To compare the previously mentioned stock market indices, daily prices and exchange rates will be taken into account, and in both cases an average will be calculated using the opening and closing values for each day.

Initially, the dates of each index price and exchange rate will be compared and only those that coincide will be kept, which will allow calculating the daily prices in USD of each of them quoted in a different currency. This approach was adopted to homogenise information and to be able to compare all markets in the same currency, as returns in domestic currency may differ from returns in foreign currency, due to exchange rate variations.

Due to the fact that each country has its own public holidays or non-trading days for its markets, the dates of all index's prices will be compared and only those on which all markets traded will be taken into account.

Once all prices expressed in USD for each of the dates have been considered, the daily returns for each country will be calculated.

With these figures it will be possible to calculate the returns and risks for each index, which will allow to analyse the behaviour of the indices, as well as to make comparisons and calculate correlations.

After this analysis, using Solver, the proportions of investment in each index will be determined in order to achieve an optimal portfolio, which will be the one with the lowest possible risk. This will allow comparisons to be made with the values obtained for the indices in isolation.

In addition, different scenarios will be determined, varying the target return, resulting in different levels of risk, to present alternative portfolios for different types of investors that can also be compared with the return of each of the indices.

2. Literature Review

Portfolio optimization and international diversification are topics that have always generated great interest and have therefore been widely studied in finance. This section briefly presents a selection of research papers that addressed the topic of international diversification, to highlight some interesting findings in this field.

As one of the most famous premises in finance states, not all the eggs should be only in one basket, diversification is a principle that everyone should respect when it comes to investing.

Several investigations had shown that an internationally diversified portfolio is less risky and more profitable than one composed only of domestic assets.

Solnik (1974), through an analysis of major European markets and NYSE stocks for the period 1966-1971 demonstrated that the total risk of a portfolio is the sum of the number of securities it includes, the individual risk of each asset and also the degree to which these risks are independent of each other.

Numerous research studies, which have included countries from different regions of the world, with different economic contexts and levels of development, have demonstrated Solnik's point and confirmed that including a larger number of assets in a portfolio leads to a reduction in risk.

An equity risk-return research on various countries such as Chile, Brazil, the US, Switzerland, Spain, Hungary, India and Japan, conducted by Chura Palacios and Ajrota Feliciano (2019), confirmed that international diversification reduces volatility and risk levels, not only systemic risk, but also allows for higher returns. According to the authors, the key is to find low levels of correlation between assets.

Loaiza Espinoza, J. (2004), also investigated this topic considering the most representatives equity indexes from different continents: America: Bovespa (Brazil), IGPA (Chile), IRECU (Ecuador), MERVAL (Argentina), DOW JONES (United States) and IPC (Mexico), from Europe: CAC-40 (France), DAX (Germany), IBEX (Spain), FTSE100 (UK), and from Asia NIKKEI (Japan). Taking a six-year sample (1998-2003) and using econometric analysis and

CAPM model, he concluded that for different risk profiles, international diversification is better than investing only in domestic markets, it does not only provide more options, but the portfolios that can be achieved are also more efficient in terms of risk and returns.

Another comparison between a domestic versus an international portfolio was made by Arredondo Ruiz (2022), who analysed how investing in Spain, Germany and France has been better in terms of return and risk than investing only in Spanish equities. The research included a sample of daily prices over a period of 2 years and 3 months, from January 2020 to April 2022, of 36 companies and applied Markowitz portfolio selection theory to perform the analysis.

As it can be seen in the above-mentioned studies, for different countries combinations and different time periods, international diversification has always demonstrated to be an efficient way to diversify a portfolio as well to reduce risk and improve returns.

But not only benefits derived from international diversification, some negative effects will be exposed in the below paragraphs. As stated by Solnik et al., (1996), when the domestic market is subject to a strong negative shock is when the benefits of international risk diversification are most needed, but increased correlation reduces this benefit. In other words, "correlation increases in bear markets but not in bull markets" (Solnik, 2001, p. 649).

A review of researches on this topic shows that on emerging countries, specifically on Mercosur countries, there is a contagion effect of volatility across countries; this paper analysed volatility behaviour, through stock return data, for a group of Asian and Latin American countries during 90s decade. They found that Latin American countries were interdependent during volatility process while they did not show dependence with Asian markets (Edwards, S. and Susmel, R., 2001).

But this is not just something that happens in less developed markets; there is also evidence of similar effects in the larger economies, where the world's most important markets take place.

King and Wadhwani (1989), analysed the contagion effect during the global stock market crash in October 1987 and confirmed that it increased during periods of high volatility.

Other authors that confirmed this behaviour are Ramchand, L., Susmel, R., (1998), Bertero and Mayer (1989), who agreed that correlations tend to increase during unstable periods, when considering major stock markets, as U.S., Japan, Germany, Canada, U.K.

Correlations are not statics, they vary over time, and this was also documented by Makridakis and Wheelwright (1974) and Bennett and Kelleher (1988), as well as Kaplanis (1988) and Koch and Koch (1991).

As it can be inferred, changes in worldwide economy such as regulations, openness of economies and integration between countries have influenced these behavioural changes in stock markets.

3. Theoretical Background

The objective of this chapter is to introduce different theoretical concepts, as well as formulas and models that will be used later on to analyse the collected data and to better understand the topic under analysis.

3.1 Technological changes, globalization and financial deregulation

Advances in communication technologies brought faster and more efficient transmission of information around the world. It reduced information gaps, making geographic distance less relevant for investors. It also influenced transactions, custody, clearing and settlement of securities. As a result, there was a reduction in transaction costs, improved liquidity in many markets and provided the necessary tools to enable stock trading around the world and across time zones (De La Torre, Schmukler, 2007).

As Ruiz Dotras, E. (2014) states, markets tend to have standardized systems for settlement and compensation of trades which turns easier to negotiate between countries. However, increased deregulation has led to greater complexity and instability, which is commonly known as volatility of financial markets.

Technological changes, as well as globalisation and financial deregulation, have caused significant changes in the international mobility of capital, with both positive and negative effects, which have modified the functioning of financial markets and the development of interactions between countries.

According to Bustelo (1999), financial globalization is understood as "the growing of mutual financial dependence among the countries of the world caused by the greater volume and variety of cross-border transactions of capital flows".

Globalisation has several effects on economies that are increasingly integrated into the world economic system, benefits can be divided into direct and indirect channels. Direct channels include the following: it helps for the development

of financial sector, facilitates the transmission of technology and knowhow from developed countries to emerging economies, reduces the cost of capital for the domestic economy through capital mobility, facilitates the accumulation of savings domestically, creates new investment and financing opportunities. On the other hand, Indirect channels include access to markets that requires improved specialization in production and allows for improved macroeconomic policies and institutions through enhanced competition (Prasad et al., 2003; Schmukler and Abraham, 2017).

However, it also involves risks which can destabilize financial markets and ultimately slow economic growth. A high degree of global integration may generate risks that do not originate directly from the domestic economy, but are transmitted from other economies (Balcilar, et al., 2019). It is called the contagion effect, and is manifested when a crisis arises in one economic area and automatically shocks other financial markets (Ruiz Dotras, E., 2014).

This transmission between countries creates both challenges and opportunities. While trade and business options are expanding rapidly, adapting to these changes requires significant institutional and productive adjustments. These adaptations are necessary to fully capitalize on the benefits of globalization while minimizing its negative effects.

The process of financial liberalisation began in the late 1960s but its relevance increased since the 1980s and was consolidated around 1990 (McKinnon, 1993; Eichengreen, 2008).

As stated by Obstfeld and Taylor (2002), from 1990 onwards, the term globalisation gained prominence to describe an increasingly integrated and interdependent world economy, with supposedly free flows of goods, services and capital.

The deregulation that has accompanied financial liberalization has often contributed to greater volatility, making some economies more susceptible to financial instability. The stability of each country's financial system is, therefore, not just a product of its own economic policies but also of its integration into the global financial ecosystem.

Flows of financial assets have outpaced the expansion of trade in goods and services. This development is supported by advances in communications and transport technology, which have allowed geographical distances to shrink and the reach and speed with which information is available to expand, resulting in faster and more efficient global financial operations (Bartram and Dufey, 2001).

As Oliver (2015) explains, it is the economic and financial structure of each country that will determine the effects of an international financial shock on the volatility of each stock market index, with this effect being greater in countries with a worse macroeconomic situation.

The key to maximizing the benefits of financial globalization lies in finding a balance-embracing technological advancements and market integration while implementing robust regulatory frameworks to manage risks. In this context, countries must adapt their economic structures and institutions to remain resilient against external shocks and ensure that the positive effects of financial globalization can be fully realized without exacerbating instability.

3.2 Asset Allocation

Asset allocation is a key aspect of the portfolio construction process and is widely considered the most important by investment professionals (Bodie et al., 2011). Although there are various ways to approach asset allocation, the ultimate decision depends on the investor's preferences and objectives.

The risk-return trade-off, determined by each investor's level of risk aversion, ultimately defines the allocation between risky assets, such as equities, and safe assets, such as bonds. This means that investors who are more risk-averse may allocate a larger proportion of their portfolio to safer assets, such as bonds, while those with a higher risk tolerance may choose to invest more heavily in equities to pursue higher potential returns.

The investment horizon is also a major factor to be considered when building an investment portfolio, this will also determine which kind of assets best suits the investor's expectations.

It is always recommended to construct a portfolio with a combination of variable and fixed return assets, however, for the purposes of this investigation the analysis will focus solely on equity investments.

3.2.1 Risk and return

This sub-section will describe some basic concepts that will be used later to further develop the theory and analyse the collected data.

Return can be defined as the change in an asset price between two moments, and it is useful to express it as percentage to make comparisons. It can be expressed as follows:

$$R_a = \frac{P_1}{P_0} - 1$$

The above equation represents the change in the price of asset a between moments 1 and 0.

To obtain the average return of a specific time period, single returns should be summed and divided by the total quantity of periods or observations under analysis (n).

$$\overline{r_a} = \frac{\sum_{i=1}^{n} r_i}{n}$$

The return of a portfolio can be expressed as the weighted average of returns of the individual assets (Bodie et al., 2011):

$$R_p = \sum_{i=1}^{n} \left(X_i * R_{ij}\right)$$

Even though the future is unknown, it is possible to forecast it. According to Olivo (2008), the expected return on a portfolio can be calculated as the sum of the proportions invested in each asset (Xi) multiplied by the expected return on each asset (E(Ri)).

$$E\left(R_p\right) = \sum_{i=1}^{n} X(i) * E(R_i)$$

In what concerns to risk, the most commonly used indicators are variance and standard deviation. Both are related, being variance the square of the expected deviation from the mean and the standard deviation the square root of the variance, commonly known as *volatility*.

According to Berk and DeMarzo (2008), variance and standard deviation of return distribution can be expressed as follows:

$$Var(R) = E[(R - E[R])^2] = \sum_R PR * (R - E[R])^2$$

$$SD(R) = \sqrt{Var(R)}$$

"The variance of a portfolio depends not only on the individual variances of the assets in the portfolio, but also on the type of joint variance between the different pairs of assets, that is the covariance" (Olivo, 2008, p.146).

The covariance is a measure of how assets move together, being positive when assets have positive and negative deviations at the same time, negative if this happens in different moments and tending to zero in case deviations are unrelated (Elton, et al., 2014).

From covariance values the correlation coefficient can be obtained, which is easier to interpretate and will be explain in the next section.

When talking about risk, it should be considered that there are two different types of it: *unsystematic, specific or diversifiable risk* that is the one that can be eliminated or reduced through diversification and the *undiversifiable or systematic risk* that is the part of the risk that cannot be eliminated even with portfolios consisting of a large number of stocks (Olivo,2008).

Every investor has a different attitude to risk and it can be classified in three main groups (Bodie et al., 2011):

- Risk averse: these kinds of investors dislike risk and they demand a higher compensation to take risk.
- Risk Neutrals: the level of risk is irrelevant for them, therefore, there is no penalty for risk.

- ❖ Risk lovers: opposite to risk averse, these investors feel comfortable when assuming risk and are willing to pay compensation for taking risk.

The tolerance level of each individual to risk will determine their investment preferences, and the compensation they will request for the risk they take.

Another commonly used indicator of risk in portfolio management is the coefficient Beta; The beta of a security is the appropriate measure of its risk, because the beta is proportional to the risk that the security contributes to the optimal risky portfolio. According to Fabozzi and Markowitz (2011, p. 101) the beta can be estimated using a regression analysis based on historical data on the observed returns for a security and the observed returns for the market. In this analysis performed by regression, security return is the "dependent variable" and market return is the "independent variable". The formula for its calculation is:

$$\beta_i = \frac{Cov\left(R_i, R_m\right)}{Variance_M}$$

Where:

- R_i = the return on an individual stock;
- R_M = the return on the overall market;
- $Cov\ (R_i, R_M)$ = how changes in a stock return are related to changes in the market return;
- $Variance_M$ = how far the market's data points spread out from their average value.

The closer the beta value is to 1, the more aggressive the investment, while stocks with betas below 1 are considered defensive.

3.2.2 Markowitz Portfolio selection Model

In 1952 Markowitz developed the modern portfolio theory also known as Markowitz Mean Variance Model, creating an optimization model for asset

allocation, which allows the selection of a portfolio that maximizes return for a given level of risk. Despite the development of other theories, all based on Markowitz's model, the mean variance model is still widely use nowadays to select the best asset allocation for each investor preferences.

To build a portfolio, returns and risks of each asset should be taken into account, using historical data of prices and determining average and standard deviation. It should be considered that the correlation level between assets will influence returns.

The model has many assumptions, being the most important that the *risks can be minimized through diversification*. Other assumptions are the following (Markowitz, 1952):

-Investors are rational and will always behave in a certain manner.

-There are only two different types of assets: low returns and high returns.

-Markowitz argues that markets will always work in a certain direction and will always be efficient.

-He also assumes that every investor has unlimited access to information about market changes.

-The last assumption is that all investors are risk-averse and that there is a bracket of bearable loss.

As it always occurs with assumptions, not all of them are applicable in the real world, however, they are necessary to build models.

To find the proportions that should be invested in each asset to minimize risk, Markowitz (1952), proposed below model:

$$Min\,\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} X_i\,X_j\sigma_i\sigma_j\rho_{ij}$$

Subject to:

- $\mathrm{E}(R\rho) = \sum_{i=1}^{n} X_i\,\mathrm{E}(R_i) = Cte$
- $\sum_{i=1}^{n} X_i = 1$

- $X_i \geq 0 (i = 1, \ \dots \ , n)$

As it is shown in the above formula, the objective is minimizing the risk of the portfolio calculated through the variance (σ_p^2) .

Covariance matrix is expressed through $\sigma_i \sigma_j \rho_{ij}$, being risk of each asset determined by its standard deviation: $\sigma j = \sqrt{\sigma_j^2}$ and $\sigma_i = \sqrt{\sigma_i^2}$

The weight of asset i in the portfolio is determined by X_i and for j by X_j.

The restrictions to minimize the objective function (variance of the portfolio) are the following:

-First of them is to give a constant value to the expected return of the portfolio $\mathrm{E}(R_\rho) = Constant$ for a determined profitability, which is the minimum risk it could be obtained.

-The second constraint is budgetary. The sum of invested weights in each asset should be equal to 1 or 100%.

-And last but not least, there is a non-negativity condition, it is not possible to invest a negative proportion, which means short selling is not allowed. In the Markowitz model there will be a fixed number of equities from which a portfolio will be built, adjusting the proportions according to risks and returns of each of them.

The key to the model is the existing *correlation* between assets in the portfolio. This is expressed in the objective function as ρ_{ij}, which lies between -1≤**ρ**≤1.

Meaning –1 a perfect negative correlation between two assets. Therefore, a positive movement in one is associated with a negative movement in the other asset. While 1 demonstrates a perfect positive correlation, that is, both assets move in the same direction in response to market movements (Berk and DeMarzo, 2008).

A perfect positive correlation increases risk of the portfolio (higher standard deviation), diversification into assets that are not perfectly correlated allows to reduce the risk of the portfolio. The lower the correlation between assets, the greater the gain in efficiency from diversification (Bodie et al., 2011).

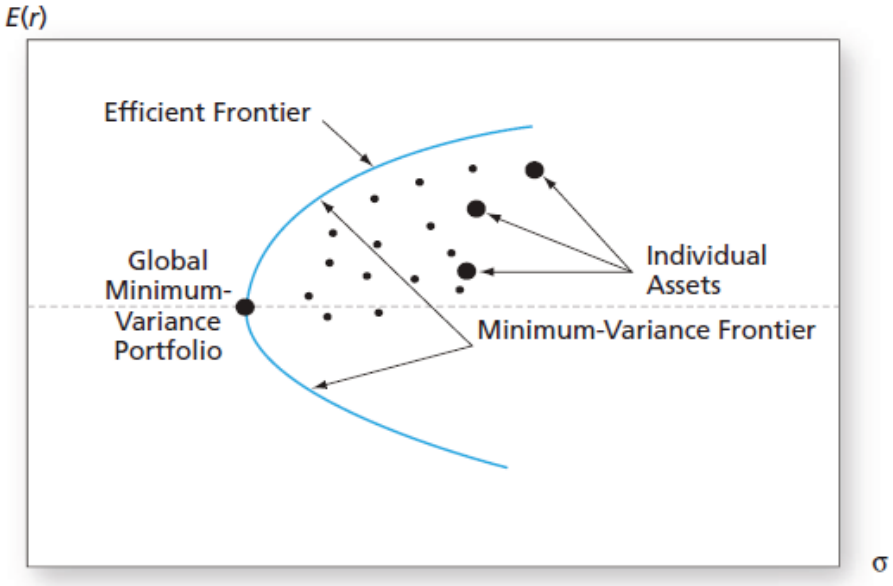
By combining different assets and the weights invested in each of them, alternative portfolios can be created. Each portfolio will have a certain risk-return ratio, which will suit different risk profiles.

All available risk-return opportunities, can be represented graphically by the *minimum variance frontier* of risky assets, which is a collection of the lowest possible variance that can be obtained for a given expected return of the portfolio (Bodie et al., 2011).

Figure 1 shows how greater expected returns (y axis) require greater expected risk (x axis).

The points to the right show that portfolios consisting of individual assets are inefficient and that diversification leads to portfolios with higher expected returns and lower risk, measured through standard deviation.

Figure 1: Efficient Frontier



Source: Bodie, et al., 2011 – Investment (p. 211)

The *efficient frontier* of risky assets is the part of the frontier that lies above the overall portfolio. For any portfolio located at the bottom of the minimum variance frontier, there is a portfolio with the same standard deviation and a higher expected return located just above it. Therefore, the lower part of the minimum variance frontier is inefficient (Bodie et al., 2011; Olivo, 2008).

3.2.3 Capital Asset Pricing Model

Known as CAPM, and based on Markowitz papers, was developed in yearly '60 by William Sharpe, John Lintner and Jan Mossin. It describes the relationship between the expected return of an asset and its risk, specifically in relation to the market.

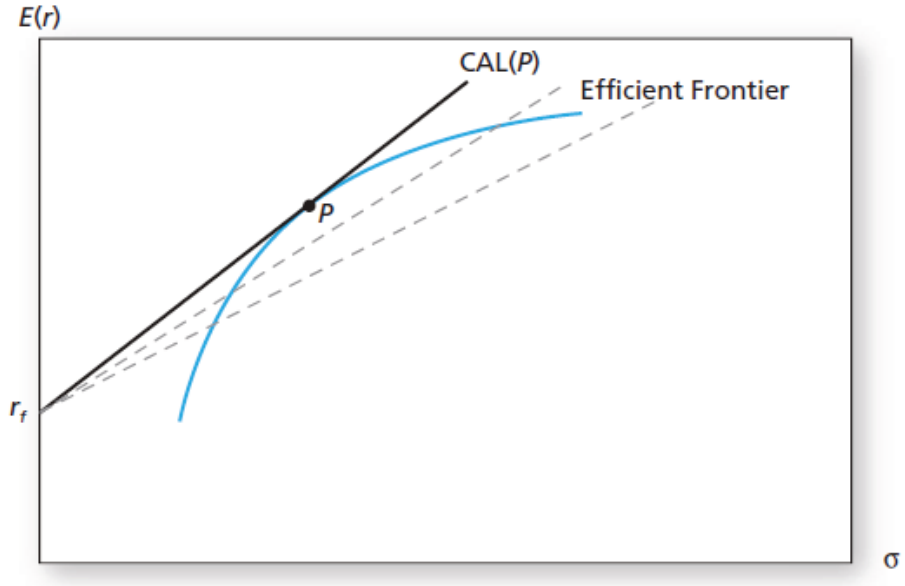
The model adds the risk-free asset to the analysis of a risky portfolio, which is an asset with a certain or known expected return, as there is no uncertainty, it has no risk and therefore its standard deviation is equal to zero.

Its three main assumptions, according to Berk and DeMarzo (2008), can be described as follows:

- Investors can buy and sell at competitive market prices (without incurring in taxes or transaction costs) and borrow or lend at the risk-free interest rate.
- Investors hold only efficient portfolios of securities that trade - those that produce the maximum expected return for a given level of volatility.
- Investors have homogeneous expectations or beliefs about the volatilities, correlations and expected returns of securities.

When including the risk-free asset, the capital allocation line (CAL) can be defined, it is a line tangent to the efficient frontier and supports the optimal portfolio P, which is the optimal risky portfolio, it maximizes the reward to volatility ratio (Bodie et al., 2011).

Figure 2: Capital Allocation Line



Source: Bodie, et al., 2011 – Investment (p. 212)

Given that risk preferences and investment objectives vary from one investor to another, the CAL will be different for all investors, defined by the individual risk-return trade-off from which they will determine the mix of risky and risk-free assets.

Considering the assumption previously mentioned, that all the investors use the same input list, all of them will hold the same optimal risky portfolio, called *market portfolio* (identify by an M on figure 3) the only difference will be the proportion invested in it and in the risk-free asset (Bodie et al., 2011).

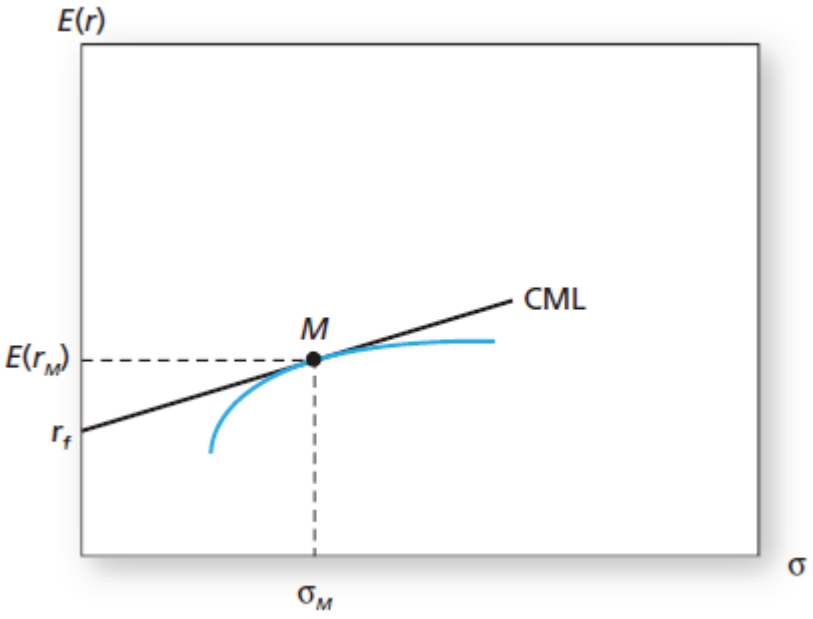
This is how the capital market line (CML) will be determined, it is the line that runs from the risk-free rate through the market portfolio, it is also the best achievable capital allocation line. The points along the CML are combinations of risk-free ad risky assets that are identical for all the investors (Elton, et al., 2014).

The tangent portfolio to the CML is called market portfolio (M), and it is the same for all the investors. The slope of the CML is the Sharpe ratio of the market portfolio, this ratio measures the reward (excess return over the risk-free rate) to volatility (standard deviation) and can be calculated as follows (Bodie et al., 2011):

$$SR = \frac{E_{(rp)} - rf}{SD}$$

The higher this ratio, the more attractive a portfolio will be, as it represents the expected excess return that can be obtained by investing in risky assets versus risk-free assets (Bodie et al., 2011; Solnik and McLeavey, 2014).

Figure 3: Efficient frontier and capital market line



Source: Bodie, et al., 2011 – Investment (p. 283)

To measure the risk that a security contributes to the optimal risky portfolio, the coefficient beta can be used, it is determined by comparing the daily returns of each stock with the daily returns of the market over the same period (Bodie et al., 2011). It measures systematic or non-diversifiable risk, usually expressed as the sensitivity to market changes.

A higher beta means more risk, but a portfolio of stocks with high beta may exist at some point in the CML where the trade-off is acceptable.

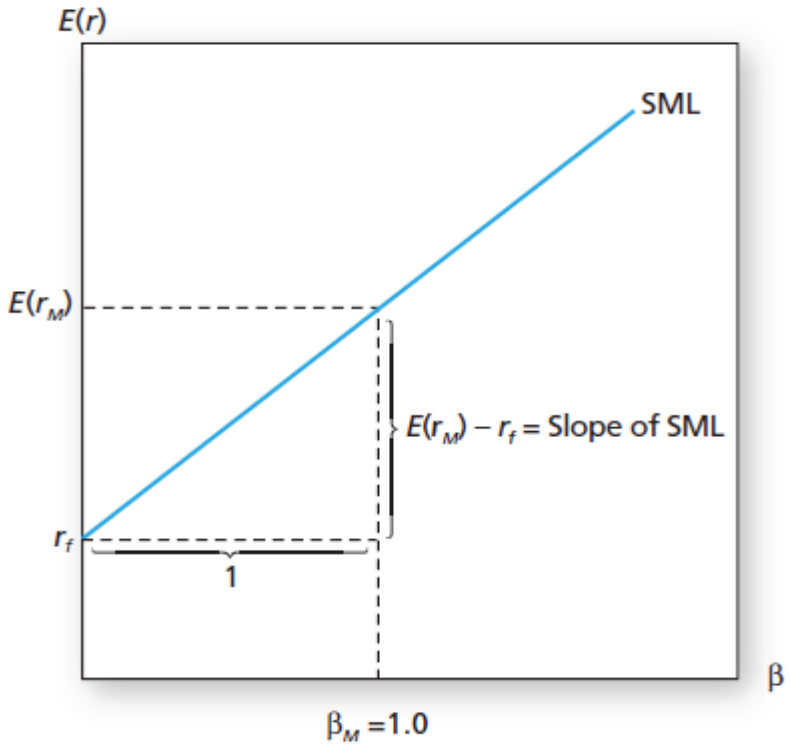
This measure can range from -1 to 1, with stocks with Beta greater than 1 being riskier than the market, while those with Beta equal to 1 are as risky as the market, and finally, stocks with Beta less than 1 are less risky than the market (Olivo, 2008).

Considering the risk-free rate and the market beta is possible to draw the SML.

The CAPM and the SML (security market line, graphically used to represent individual asset risk premiums as a function of asset risk) establish a relationship between a stock's beta and its expected risk.

According to the CAPM, the market portfolio is efficient and equivalent to the required return which is equal to the expected return for each security. Therefore, all securities and portfolios must be in the SML (Berk and DeMarzo, 2008).

Figure 4: Security market line



Source: Bodie, et al., 2011 – Investment (p. 289)

In the figure, the slope equal to 1 represents the market beta, i.e. the risk premium of the market portfolio, while the vertical axis reflects the expected return of this portfolio.

"It is useful to compare the SML with the CML. The CML graphs the risk premiums of efficient portfolios as a function of portfolio standard deviation. In contrast, the SML graphs individual asset risk premiums as a function of asset risk" (Bodie et al., 2011, p. 289).

The CAPM formula is expressed as:

$$E(r_i) = r_f + \beta_i\left[E(r_M) - r_f\right]$$

Where:

- $E(r_i)$ is the expected return of the investment.
- r_f is the risk-free rate.
- β_i is the beta of the investment.
- $E(r_M)$ is the expected return of the market.

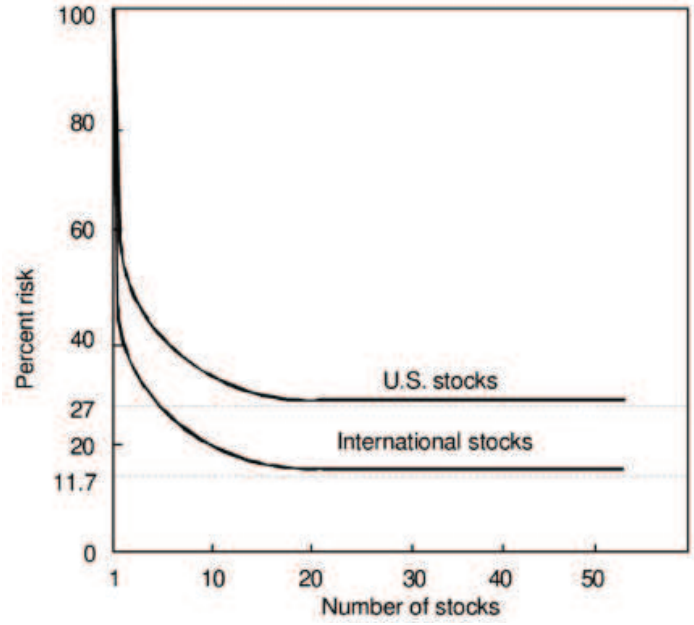
3.3 International Diversification

Globalization has, among other things, broadened investment possibilities by facilitating access to international markets. As a consequence, it has increased international investment and improved the management of asset portfolios through new combinations of return and risk.

Solnik (1974), carried out an empirical analysis in which he concluded that by expanding investment possibilities, portfolio risk can be reduced.

In the image below, Solnik (1974) concluded that specific risk could be eliminated to some extent by introducing a considerable number of assets into the investment portfolio.

Figure 5: international diversification



Source: Solnik, 1974. Why not diversified internationally rather than domestically?. PP. 48-54

The graphical representation is a comparison between a domestic portfolio (USA) and an international portfolio, in both of which a reduction in risk is observed as more stocks are introduced, with a preponderance of this effect in the international portfolio.

The remaining risk is specific risk, what means that it depends on the characteristics of the company itself and cannot be completely reduced (Berk and DeMarzo, 2008). This can be seen in the graph, by stopping the risk reduction from a given number of actions.

"The effectiveness of diversification in reducing the risk of a portfolio varies from country to country" (Elton, et al., 2014, p. 59). International diversification is justified by the low correlation between markets. Market integration is an important factor affecting the size of correlation. Further integration of domestic markets may mean that investors will have to invest in foreign markets to obtain the benefits of international diversification (Elton et al., 2014).

Although Solnik demonstrated how international diversification can contribute to risk reduction, risk cannot be completely eliminated, because of non-diversifiable or firm-specific risk.

3.3.1 International Asset Pricing Model

The International Asset Pricing Model (IAPM) extends the Capital Asset Pricing Model (CAPM) to a global context, accounting for the unique risks associated with investing in international markets.

The papers of Grauer, Litzenberger and Stehle (1976) and, above all, Solnik (1974) proposed a first international asset pricing model, incorporating a new risk, exchange rate risk. Solnik's model basically proposes that the risk premium of an asset over its risk-free domestic security is proportional to the risk premium of a world equity portfolio (Gómez-Bezares, et al. 2004).

Solnik (1974), expressed this model as follows:

$$E(R_i) = R_{Fi} + \beta_{iw}[E(R_{MW}) - R_{FW}]$$

Where:

- $E(R_i)$ is the expected return on assets index of country i
- R_{Fi} is the risk-free return of domestic country i
- $E(R_{MW})$ is the expected return of international market
- R_{FW} is the risk-free return of international market

Risk is expressed through beta, β_{iw}, which measures international systematic risk of country i.

As stated by Solnik (1974), each stock is indirectly influenced by the world market through its home market. Therefore, a link between betas in CAPM and IAPM can be established:

$$\beta_{iw} = \beta_i \times \beta_{pw}$$

β_{iw} stands for international systematic risk of country i. β_i measures systematic risk for domestic market and β_{pw} expresses the sensitivity of the domestic market factor to the world market factor.

IAPM is basically and adaptation of CAPM, considering world risk and return and adding them to the CAPM expression.

3.4 Financial vehicles for investing abroad

International investment can be done in different ways, it can be directly, by purchasing assets from other countries or indirectly, using different available financial instruments such as depositary receipts (DRs), mutual funds or exchange traded funds (ETF's).

The option to purchase securities directly in capital markets of other countries is mainly used by large institutional investors but is not very common for small investors, as there are many instruments that allow small investors to do so more efficiently.

The DR represents a security issued by a domestic institution for a foreign security held in trust in its name abroad, it allows investors to access to foreign market assets. The depositary, through the issuance of these securities, transforms the securities from the original market into instruments appropriate to the legal environment of the investor's market (Bartram and Dufey, 2001).

These programs exist in several countries, however, in the US is very well known, the name it receives is ADR (American depository receipt), a certificate that represents equities of a foreign market, owned and issued by an American bank. Through this instrument foreign companies can be traded on US markets (Khan, 2019; Solnik and McLeavey, 2014).

Another option are mutual funds, which are a great alternative to invest in a diversify portfolio only with a small amount of money. "They pool investors' money and invest it in specific assets to achieve a stated investment objective. Investment funds are offered to the public at a quoted price, which reflects the market value of the fund's assets" (Solnik and McLeavey, 2014, p. 529).

These portfolios are managed by professionals and since there are multiple types of them, each investor can select the fund that best suits his objectives and expectations (Khan, 2019) and also several of them have an international focus, which allow investors to access to an internationally diversified portfolio. Due to these benefits, Bartram and Dufey (2001), consider them as the easiest and most effective way of investing abroad.

Regarding ETFs, Khan (2019) explains, they are a special type of fund, that it is traded in stock exchanges. These funds can hold different instruments such as equities, commodities, bonds, etcetera. Like mutual funds it also guarantees access to a diversify portfolio managed by professionals. Their objective generally is to track closely the performance of a specific index.

As stated by Solnik and McLeavey (2014), "ETFs have a management cost advantage over traditional mutual funds because there is no shareholder accounting at the fund level" (p. 208).

Going further, to more complex instruments, derivates appear as another investment option. According to Solnik and McLeavey (2014), they consist in underlying assets, and to invest on them, less than the price of the underlying asset is required, it is a leveraged investment, that allow investors to multiply the rate of return. They can be used to take advantage of a specific profit opportunity or to hedge a portfolio against different risks.

It is possible to trade options and futures on different stock indices traded on different stock exchanges (Bodie et al., 2011). Derivatives also exist in the form of private contracts between two parties, in this case can be forward or swap contracts (Solnik and McLeavey, 2014).

Each investor will opt for one or another instrument depending on their investment requirements, objectives, risk profile, investment horizon and experience, because, as it will be explained below, there are pros and cons of investing abroad and all of them should be considered when making a decision and selecting the vehicle or combination of them that best suits each case.

The number of options is diverse and also the combinations that can be made with them; there are even more alternatives, but here the intention was to introduce the most common ones that are available in almost every country.

3.4.1 Advantages and disadvantages of international investment

Investing internationally can be both beneficial and challenging due to the different scenarios that can arise. This section will discuss the main benefits and difficulties of investing abroad.

The following can be considered as attractions of international investment: participation in the growth of other markets, hedge of the investor's consumption basket, diversification effects and possibly abnormal returns due to market segmentation.

Investing internationally can undoubtedly enhance a portfolio's potential, particularly through diversification, exposure to different growth opportunities, and the ability to hedge against risks inherent in a single country's economy. But there are also negative aspects, that are mentioned in the following paragraphs.

"It is claimed that international markets are not integrated but segmented because of various impediments to capital mobility. Although each national market might be efficient, numerous factors might prevent international capital flows from taking advantage of relative mispricing among countries" (Solnik and McLeavey, 2014., p. 134).

Transaction costs is one of the main barriers when considering to invest abroad, even though it varies depending on the country, usually it is more expensive to invest abroad than in the investor's country. In developed countries these costs tend to be lower than in emerging economies, where sometimes it can also be hard to access the stock market mainly due to capital account restrictions (Khan, 2019). Limitations also include the access to information, management fees and custodial services (Solnik and McLeavey, 2014).

Another item to be considered is the foreign exchange risk when investing in a country with a different currency. Nowadays there are instruments like derivatives that can be used to hedge investment against it, but it implies an additional cost that should be considered. Exchange controls is also an

obstacle that can appear in some countries, often making it difficult to withdraw/enter money and convert it to another currency.

Political and liquidity factors must be considered as well, especially when investing in small and/or emerging countries, where these risks are higher. Political risk includes default risk due to government actions and general uncertainty about political and economic developments (Bartram and Dufey, 2001). Regarding liquidity, traded volumes can be much lower than in developed countries due to lack of liquidity and more restrictive hours to invest can be faced as well (Khan, 2019).

Beside the above-mentioned, Bartram and Dufey (2001), also considered obstacles like taxation which can be discriminatory, due to heavier tax for foreign investments or even the occurrence of double taxation (Solnik and McLeavey, 2014); other obstacles are: capital market specific regulations and controls, legal restrictions.

And last but not least, Solnik and McLeavey (2014) included some other factors as psychological barriers, described as unfamiliarity with foreign markets, language and sources of information. This is why investors often prefer to invest in its own country rather than internationally.

These intangible obstacles often lead investors to prefer domestic investments, as they are perceived to be more understandable and manageable. Despite the growth of international financial markets, this "home bias" is still a common phenomenon in the investment community.

In less developed countries the access to information about economic activity and its future forecast is harder, what is a fundamental input for investment decisions.

While international investing can be a powerful tool for portfolio diversification and capital growth, it requires careful consideration of various factors. The trade-off between risk and reward is more complex than it may appear, and investors need to be aware of both the potential for higher returns and the significant challenges they may face in terms of transaction costs, risk management, and market accessibility.

3.5 Stock market indices

This section is intended to explain briefly the selected indices[1] that will be used in the next section to analyse the efficiency of internationally diversified equity portfolios, which is the main objective of the research.

As stated by Solnik and McLeavey (2014), stock market indices allow us to measure the average performance of a national market. There are different types of them, depending on the method used for its calculation. There are broadly based, market value-weighted indices, where each company is assigned an index weight proportional to its market capitalization. Another type are equal-weighted indices, where each company is assigned an index weight proportional to its market price.

Lately new indices have been introduced, for example, those based on a small number of large stocks as well as emerging, regional and global indices. In this way, different objectives are meet and different audiences are covered.

It is widely known that capital market activity has been concentrated in a few large international financial centres, mainly Frankfurt, London, New York and Tokyo. While these markets are covered in this analysis, it was decided to also consider two South American indices, which although less representative at global level, both are important for the region and above all, to analyse their relationship with the rest of the markets.

Although Latin American stock markets have grown substantially since 1990, the region's capital markets remain underdeveloped compared to markets in industrialised countries and East Asia. Equity markets in Latin America are highly concentrated, with a small number of companies accounting for a large percentage of traded volume, yet the region's derivatives markets are indeed large (De La Torre, Schmukler, 2007).

On the other hand, U.S. securities markets are the largest in the world and they also have the best reputation from a "technical" point of view: they are generally well regulated, and are characterized by breadth, depth, and resilience. U.S. investors still hold only a small number of foreign securities, in

[1] The portfolio composition for each index can be found in Appendix A.

contrast to foreign holdings of U.S. securities which are about twice as large (Bartram and Dufey, 2001).

In the below figure a graphical comparison between the biggest world stock markets, as of end of 2020, is shown. It presents the market capitalisation and number of listed companies, and the bubble size represents their share in global market capitalisation.

It is clear that the US has the largest share, followed by Europe in second place and China in third, while Japan is a little further behind in fourth place.

Figure 6: Number of listed companies and market capitalisation

Source: OECD, 2021, p. 51. Based on: OECD Capital Market Series dataset, FactSet, Thomson Reuters Eikon, Bloomberg

Since different regions and countries will be considered, using World Bank[2] overview and Open Data as common source for all of them, some figures will be presented to characterized countries' economies and show the differences between them.

[2] https://www.worldbank.org/

- ***Brazil***

According to World Bank, this big country in south America, known as the world's third-largest agricultural and food exporter, had in 2022 a population of ca. 215 million inhabitants of which 23.5% lived in poverty. Real GDP per capita was of US$9,032 in 2023 and is projected to grow at approximately 2.3 percent over the medium term. Inflation of 4.6% in 2023, is projected to gradually converge towards 3.8 percent by 2025, within the Central Bank's target range.

Brazil's macroeconomic buffers remain solid, with ample international reserves, low external debt, a credible and independent central bank, a resilient financial system and exchange rate flexibility. In addition, Brazil initiated the first phase of a long-awaited reform of indirect taxes, which is expected to improve the business environment through tax simplification and to boost productivity.

However, the general government primary deficit reached 2.4% of GDP in 2023, from a surplus of 1.2% in 2022, while public debt stood at 74.4% of GDP, up from 71.7% in 2022.

Brazilian stock exchange index is called Bovespa[3] (Bolsa de Valores do Estado de São Paulo), it is the main performance indicator for traded shares and brings together the most important companies on the Brazilian capital market. It was created in 1968 and, over the course of 50 years, has become a benchmark for investors around the world.

Revalued every four months, the index is the result of a theoretical portfolio of assets. It is made up of the shares and units of companies listed on the Bovespa that meet the criteria described in its methodology, corresponding to around 80% of the number of trades and the financial volume of the capital market.

To be selected, companies must fulfil the following criteria: be among the assets that represent 85% in descending order of the Tradability Index (IN) (90% buffer); 95% presence on the trading floor; 0.1% of the financial volume on the spot market (standard lot); and not be a penny stock[4].

[3] Information about Bovespa was retrieved from https://www.b3.com.br/pt_br/market-data-e-indices/indices/indices-amplos/ibovespa.htm

[4] Given name to the stock of a small company that trades for less than $5 per share.

Weighting: Market value of free float/cap 20% per company/cap 2x IN. Rebalancing: Every four months (1st Monday of January, May and September).

- ***Mexico***

With nearly 130 million inhabitants is the second largest economy in Latin America and the 15th worldwide. Poverty rate reached 36.3% in 2022. In 2023 GDP was USD 1.79 Trillions, it means USD 13,926 per capita, a 3.2% growth from the previous year. Inflation rate was 5.5% in 2023, a decreased of 2.4% from 2022.

It is characterized for having abundant natural resources and strong macroeconomic institutions, an open trade policy, and a diversified manufacturing base integrated into global value chains.

Mexican Stock Exchange[5] (BMV) is a fully integrated Exchange Group that operates cash, listed derivatives and OTC markets for multiple asset classes, including equities, fixed income and exchange traded funds, as well as custody, clearing and settlement facilities and data products for the local and international financial community.

BMV is the second largest stock exchange in Latin America with a total market capitalization of over US$ 530 billion.

The index S&P/BMV IPC, composed of 35 companies, aims to measure the performance of the largest and most liquid stocks listed on the Mexican Stock Exchange. The components of the index are listed by modified market capitalisation, subject to diversification requirements. The index is rebalanced twice a year, in March and September.

- ***United Kingdom***

Comprising England, Scotland, Wales and Northern Ireland, is an island nation in north-western Europe with a total population of 68.35 Mio in 2023.

[5] Information about Mexican market as retrieved from its official website: https://www.bmv.com.mx/ and also factsheet of the index, additional information as well as methodology from https://www.spglobal.com/spdji/es/indices/equity/sp-bmv-ipc/#overview

It is the world's sixth largest economy and a key player in the global economy, with a major role in financial services; another important sources of income for this world-leading economy are tourism, manufacturing and retail trade, and financial services. Its GDP in 2023 was USD 3.34 trillion and its GDP per capita reached USD 48.866, while inflation rate for this year was 6.8%.

FTSE 100 is the benchmark stock market index of the London Stock Exchange[6]. It is composed of the 100 companies with the largest market capitalisation in the United Kingdom, with the companies comprising the index accounting for 70 % of the total value of the London stock market.

The index is maintained by FTSE Group, a subsidiary of the London Stock Exchange Group. Founded on 3 January 1984, its acronym stands for Financial Times Stock Exchange 100, and it is effectively controlled by the Financial Times magazine.

Stocks are capitalisation-weighted and are reviewed quarterly, on the first Friday of March, June, September and December.

- ***United States***

It is a highly developed mixed economy, with a population in 2023 of 334.9 million. It is the world's largest economy by nominal GDP, equivalent to USD 27.36 trillion in 2023; it is also the second largest by purchasing power parity (PPP), behind China. It has the sixth highest GDP per capita (nominal) in the world, estimated at USD 81,700 in 2023. In the same year the inflation rate reached 4.5%.

The S&P500[7] is considered the best indicator of US large-cap stocks. It includes 500 leading companies listed on the NYSE and NASDAQ exchanges and represents approximately 80% of the available market capitalisation in the US and approximately 50% of the global equity market.

[6] Official website was used to retrieve information: https://www.londonstockexchange.com/

[7] Information retrieved from https://www.spglobal.com/spdji/es/indices/equity/sp-500/#overview (factsheet, methodology and additional information), and also from S&P500, available on: brochure https://www.spglobal.com/spdji/es/documents/additional-material/sp-500-brochure-es.pdf

This index is weighted by free float-adjusted market capitalisation, which means that its weights reflect the full range of investor expectations. It is rebalanced on a quarterly basis in March, June, September and December. Changes over time in both the five largest stocks and the GICS sector weights in the index show how the composition of the index has evolved to reflect the increased importance of IT companies, and the decreased importance of companies in the Industrials and Energy sectors. Being in 2023 the top five components of the index Apple Inc., Microsoft Corp., Amazon.com Inc., Nvidia Corp. and Alphabet Inc. A

- ***European Union***

Composed by 27 countries, with a total population of 449.5 million inhabitants representing 5.6% of the world's population. It is one of the world's biggest economies with a GDP of USD 18.35 trillions in 2023, having Germany the largest share, followed by France and Italy. GDP per capita reached USD 40,824 and inflation rate was 6.3% during the same year.

The European Union consists of an internal market of mixed economies based on free markets and advanced social models. The member countries form a single internal market with free movement of goods, services, capital and labour.

Euro Stoxx 50[8] is derived from the EURO STOXX index and represents the performance of the 50 largest companies among the 20 super sectors in terms of free-float market capitalization in the Eurozone. The index has a fixed number of components and is part of the STOXX blue-chip index family. The index captures about 60% of the free-float market cap of the EURO STOXX Total Market Index (TMI).

The weighting by country is distributed as follows: 39.5% France, 26.7% Germany, 14.7% Netherlands, 8.2% Italy, 7.6% Spain, 1.7% Finland and 1.6% Belgium.

[8] Information retrieved from https://stoxx.com/index/sx5gu/?factsheet=true as well as additional information available on the official website.

The methodology for the construction of the index is described as follows in the factsheet of the index, available on the official website: from each of the 20 EURO STOXX Supersector indices, the largest stocks are added to the selection list until the coverage is as close as possible to 60% of the free float market capitalisation of the corresponding EURO STOXX TMI Supersector index. In addition, all current EURO STOXX 50 stocks are added to the selection list.

The 40 largest stocks from the selection list in terms of free float market capitalisation are selected; the remaining 10 stocks are selected from the largest current stocks ranked between 41 and 60; if the number of selected stocks is still below 50, the largest remaining stocks are selected until there are 50 stocks.

- ***Japan***

Is the only Asian country selected in this research, it can be described as a highly developed mixed economy. It is the fourth-largest economy in the world by nominal GDP behind the United States, China, and Germany (Asahi Shimbun, 2024).

It had 124.5 million inhabitants in 2023 and a GDP of USD 4.21 trillions, which represents a GDP per capita of USD 33,834 for the same year. Inflation rate in 2023 was 3.3%.

The Nikkei 225[9] is a price-weighted equity index, which consists of 225 stocks in the Prime Market of the Tokyo Stock Exchange. The selection is done based on two factors: liquidity and sector balance, the index introduces 6 sectors categories consolidated from the 36 Nikkei industrial classifications.

The index covers representative companies from major industries. It weights highly on technology sector which deploys its business globally and is sensitive to the change in world economic climate. It is reviewed every six months, in April and October.

[9] Information retrieved from https://indexes.nikkei.co.jp/en/nkave/index/profile?idx=nk225

4. Analysis of collected data

This chapter will present an analysis for the collected data using the theory exposed in chapter 3 to explain the findings and draw the results.

4.1 Return and risk analysis

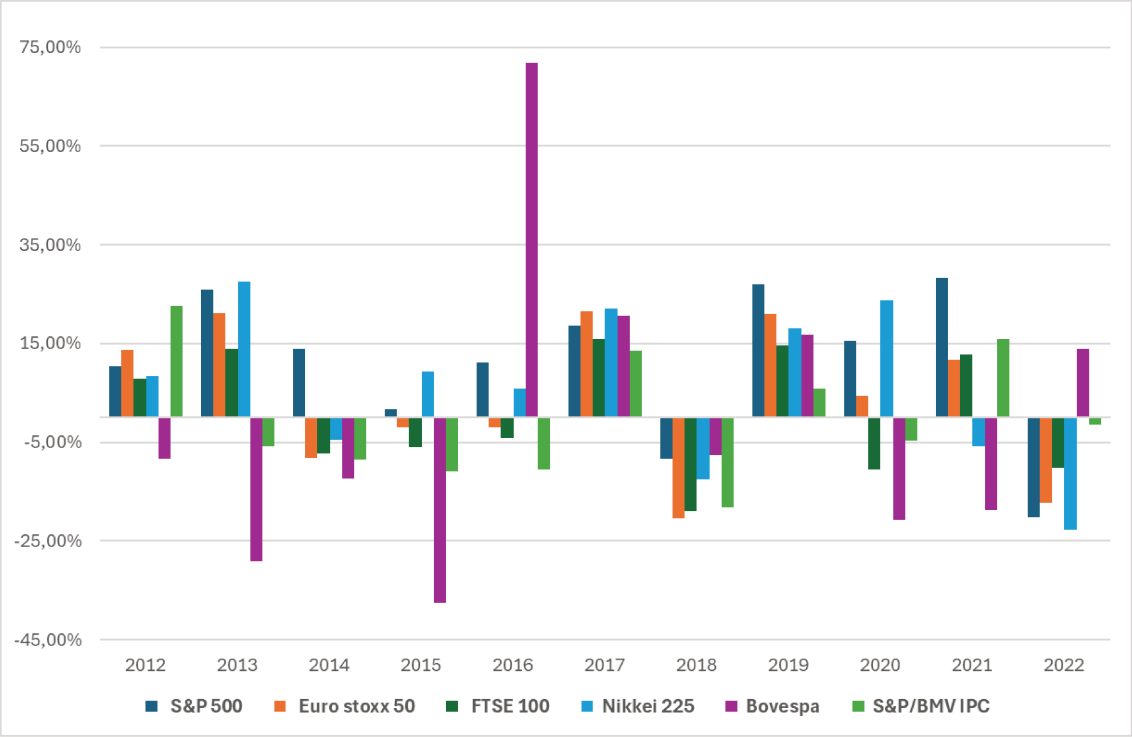
After following the steps explained in section 1.3.3 to homogenise data, 2.404 observations for each index were left, corresponding each of them to a date where all six indices traded. As it was mentioned before, all the data will be used in USD for all of them.

As a first step, a comparison of annual returns for the whole period under analysis and for all the indices is shown below[10].

Looking at the evolution over time is useful to have a first overview of the data to be used. There are only two years in which all indices had positive returns, 2017 and 2019, while only one year, 2018, in which all indices were negative. For all other years the returns were mixed.

[10] The complete table with all annual yields can be found in the appendix B.

Graph 1: Annual Yields



Source: own creation based on data retrieved from Investing.com

Going into further details, it can be observed that the S&P 500 shows positive returns in most of the years, with negative returns only in 2018 and 2022, while the Bovespa index shows the largest variation, with an incredible positive return in 2016 and steep losses in 2013 and 2015.

The S&P/BMV IPC is the index with the highest number of years with negative returns, 7 out of 11. The best performance was achieved in 2012 with 23% followed by 2021 with 15,86%.

Between the Euro Stoxx 50 and the FTSE 100, a similar behaviour is shown for the whole period, in terms of gains and losses. In both cases the best year was 2017, followed by 2019 and 2013 and the worst years were 2018 and 2022 for both; while 2020 was a year of significant decline for the FTSE 100 it was a year of positive performance for the European index.

The Nikkei 225 has only 4 years with negative returns, its best year being 2013, 2020 and 2017 while its biggest losses were recorded in 2022 and 2018.

The table below presents the average returns for the whole period, with the best performers being the S&P 500 and the Nikkei 225, the European index appears in third place, while the FTSE 100 is the fourth with a slightly positive average return. On the other hand, the two Latin American indices show negative figures.

Table 1: Average returns for the period 2012-2022

	S&P 500	Euro Stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
Average	11,24%	3,98%	0,73%	6,28%	-1,02%	-0,19%

Source: own creation based on data retrieved from Investing.com

Using daily prices, returns were obtained for each day, from which risk indicators were calculated for each index. In addition to the average returns, variances and standard deviations were obtained.

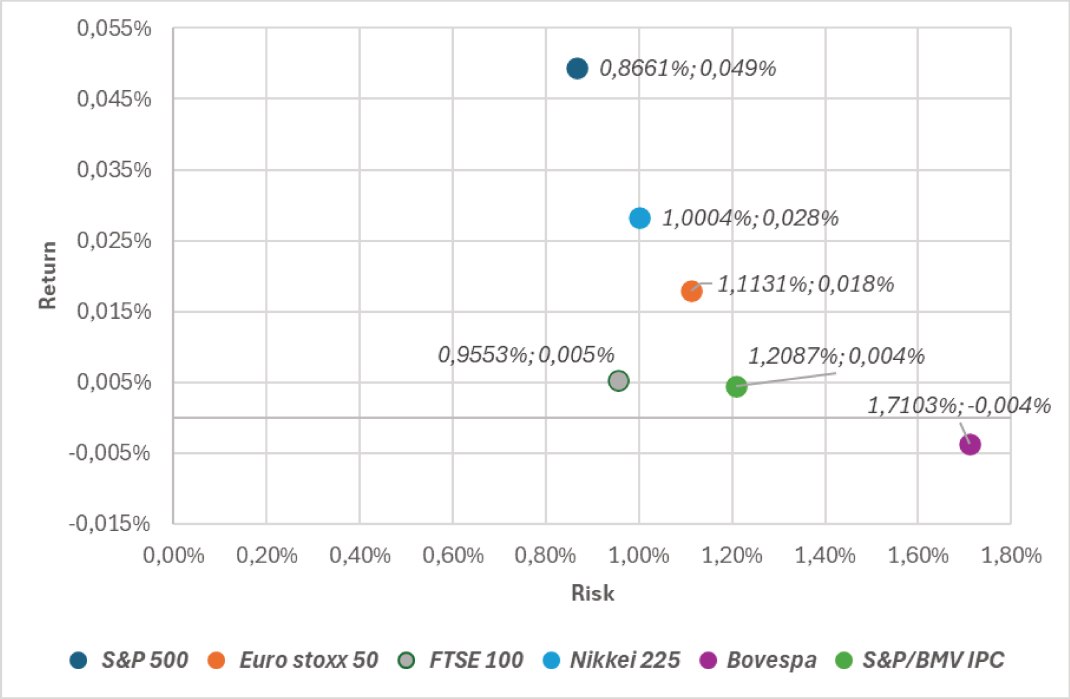
These indicators allow the observation of the risk-return relationship, to know the volatility and expected return of each index. Graph 2[11] shows, for each of the market indices under analysis, what the return-risk trade-off would be if 100% of the portfolio were invested in only one index.

The least volatile in order from left to right are S&P 500, FTSE 100, Nikkei 225, Euro Stoxx 50, S&P/BMV IPC and Bovespa.

In terms of performance, the US index is the leader, with Japan in second place, then Europe, the FTSE 100 in fourth place and the Mexican index at almost the same level, with the Brazilian index in last place and with a considerably lower performance than the rest.

[11] Details are shown in appendix B.

Graph 2: Average risk-return trade-off of each index



Source: own creation based on data retrieved from Investing.com

4.2 How are indices related to each other?

Although with the information provided by the above graphs it is possible to intuit how the indices move together, as explained in the previous section, there are specific measures to analyse this, such as covariance and correlation.

The following tables present both measures that allow us to deduce which indices are more and less related to each other, i.e. how they move in response to a variation in the other.

In the covariance matrix it is possible to see that all values are positive although very close to zero. Positive means that they move together in the same direction, when one index goes up so does the other; while a value of zero indicates that the variables are not related.

Table 2: Covariance Matrix

COVARIANCE MATRIX	USA	Europe	United Kingdom	Japan	Brazil	Mexico
cov(x,y)	S&P 500	Euro Stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
S&P 500	**0,000075**	0,000064	0,000052	0,000043	0,000072	0,000064
Euro stoxx 50	0,000064	**0,000124**	0,000090	0,000065	0,000096	0,000080
FTSE 100	0,000052	0,000090	**0,000091**	0,000054	0,000087	0,000071
Nikkei 225	0,000043	0,000065	0,000054	**0,000100**	0,000062	0,000053
Bovespa	0,000072	0,000096	0,000087	0,000062	**0,000293**	0,000127
S&P/BMV IPC	0,000064	0,000080	0,000071	0,000053	0,000127	**0,000146**

Source: own creation based on data retrieved from Investing.com

It is possible to get a better picture of the relationship between indices through the correlation coefficient, whose values vary between -1 and 1, where -1 is a perfect negative correlation that exists in cases where they move inversely and 1 is a positive correlation that exists in cases where they move in the same direction.

In cases of a perfect positive correlation, there is no possibility of diversification to reduce risk, whereas the lower and further from zero the coefficient is, the more efficiency that can be obtained through diversification.

Table 3: Correlation Matrix

CORRELATION MATRIX	USA	Europe	United Kingdom	Japan	Brazil	Mexico
correl(x,y)	S&P 500	Euro Stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
S&P 500	**1,00000**	0,66814	0,62875	0,49715	0,48378	0,61307
Euro stoxx 50	0,66814	**1,00000**	0,84930	0,58474	0,50332	0,59287
FTSE 100	0,62875	0,84930	**1,00000**	0,57001	0,53080	0,61526
Nikkei 225	0,49715	0,58474	0,57001	**1,00000**	0,36154	0,43675
Bovespa	0,48378	0,50332	0,53080	0,36154	**1,00000**	0,61388
S&P/BMV IPC	0,61307	0,59287	0,61526	0,43675	0,61388	**1,00000**

Source: own creation based on data retrieved from Investing.com

In the correlation matrix, it is easier to observe how the indices move together, being the strongest relationships (values closer to 1) the ones between Euro Stoxx 50 and FTSE 100, S&P 500 and Euro Stoxx 50, S&P 500 and FTSE 100 and a little less correlated but still over 0.5 can be found S&P 500 and S&P/BMV IPC and S&P/BMV IPC and Bovespa.

On the other hand, the less correlated are Japanese and Brazilian indices, Japanese and Mexican indices, as well as Brazilian and American indices and American and Japanese, indicating these combinations possible options for portfolio diversification.

Up to this point, the risk-return analysis was based on the individual indices and the last two matrices showed the co-movements of the indices. In this way, it has been possible to see the behaviour of the indices over time as well as the relationship between them.

The next step will be to analyse them together, being part of a portfolio in order to subsequently, through the use of optimisation tools, try to construct the optimal portfolio, which will be carried out by combining the indices in such a way as to achieve the lowest possible risk.

Later, applying different investor profiles, different portfolios will be created and the risk-return ratio of each one of them will be analysed.

4.3 Portfolio Analysis

By applying the Markowitz mean-variance model, it is possible to create a portfolio with minimum risk by combining all the investment possibilities, which in this case are the six stock market indices.

Before reaching this point, it is important to analyse what happens if the same proportion is invested in each of them, meaning 16.67% invested in each index, to obtain a total of 100% (to respect the restriction of the model).

Taking these investment proportions into account, it is possible to calculate the risk and return of the portfolio and compare them with the above figures for each index on an individual basis.

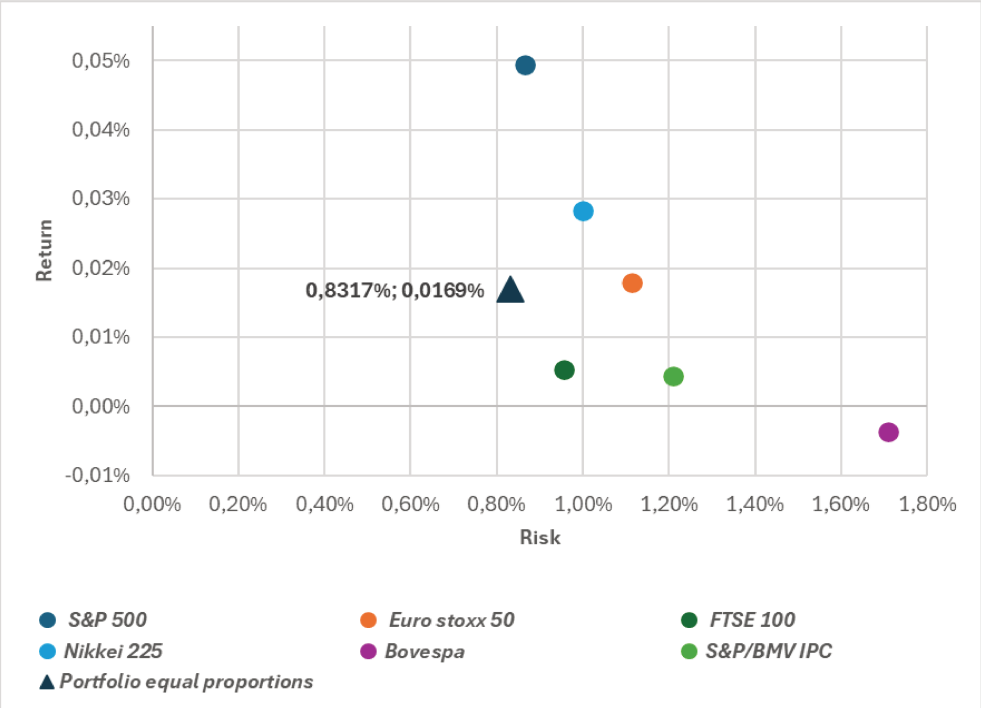
The return was calculated considering the proportion invested in each index multiplied by the average return of each of them, which were obtained using the historical daily prices.

Then the variance was calculated applying the formula for it; as a first step the difference between the return of each index to the average of it was considered, then, the square of each value was calculated and after that the sum of all of them was obtained, finally, this number was divided by the total observations.

From the variance, the value of the standard deviation was reached by applying the square root to the variance.

The graph below shows where the portfolio sits in the space previously used to represent the risk-return relationship of the individual indices.

Graph 3: Portfolio of equal proportions in comparison to indices in terms of return and risk



Source: own creation based on data retrieved from Investing.com

By investing the same proportion in each index, a portfolio with a return of 0.017% and a risk of 0.83% is achieved, which is slightly lower risk than the S&P 500 index (and significantly less than the risk of investing 100% in any of the other indices) while in terms of return the portfolio is outperformed only by three indices: slightly by the Euro Stoxx 50 and to a greater extent by the Nikkei 225 and the S&P 500.

Following Markowitz, the objective of the model he developed is to minimize the risk, therefore, using Solver and from the historical data, the optimal portfolio was created, and it is the following:

Table 4: Composition of the optimal portfolio (min. Variance)

S&P 500	Euro stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
50,76%	0,00%	21,74%	27,50%	0,00%	0,00%

Source: own creation based on data historical data and using Solver

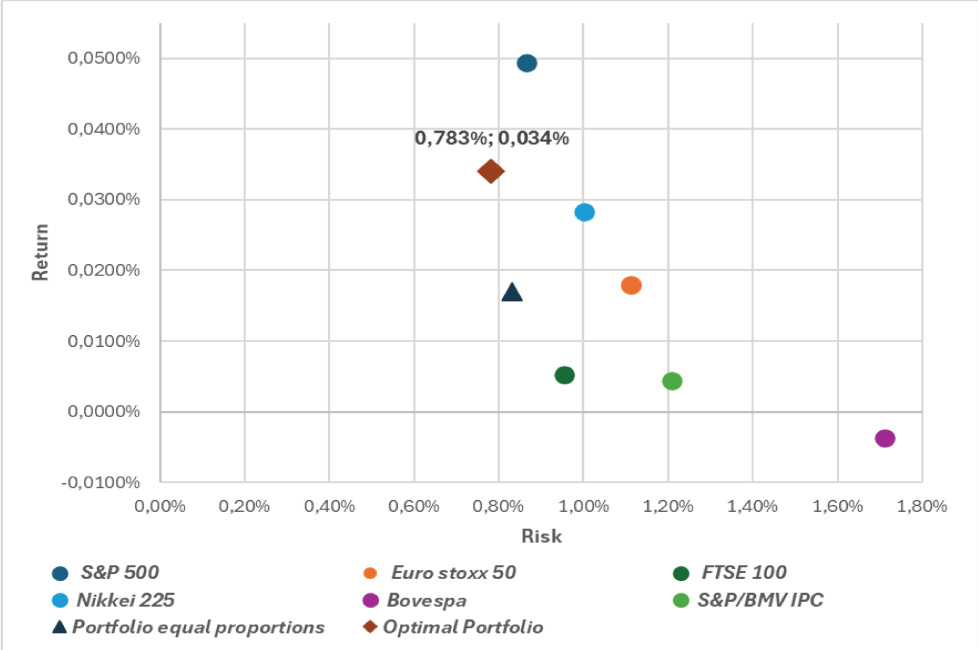
- The optimal portfolio has a *return of 0.034%* and a *risk of 0.783%.*

The minimum variance portfolio is composed of only three indices: S&P 500 with more than 50% invested in it, then the Japanese index and with a smaller proportion the FTSE 100 index.

The system does not suggest investing in the other three indices in order to minimise risk.

In the graph is shown where the optimal portfolio would be allocated, reducing the risk that was obtained in the previous portfolio and also the ones from the individual indices. In terms of return, the only portfolio that outperformed the optimal one is the S&P 500 index.

Graph 4: Optimal Portfolio (minimum variance)



Source: own creation based on data historical data and using Solver

If the objective is to maximise returns, the solution offered by Solver is to invest 100% in a single index, the S&P 500. Given the results obtained previously, this was a predictable suggestion, as it offers the highest return with the lowest risk among all the indices analysed.

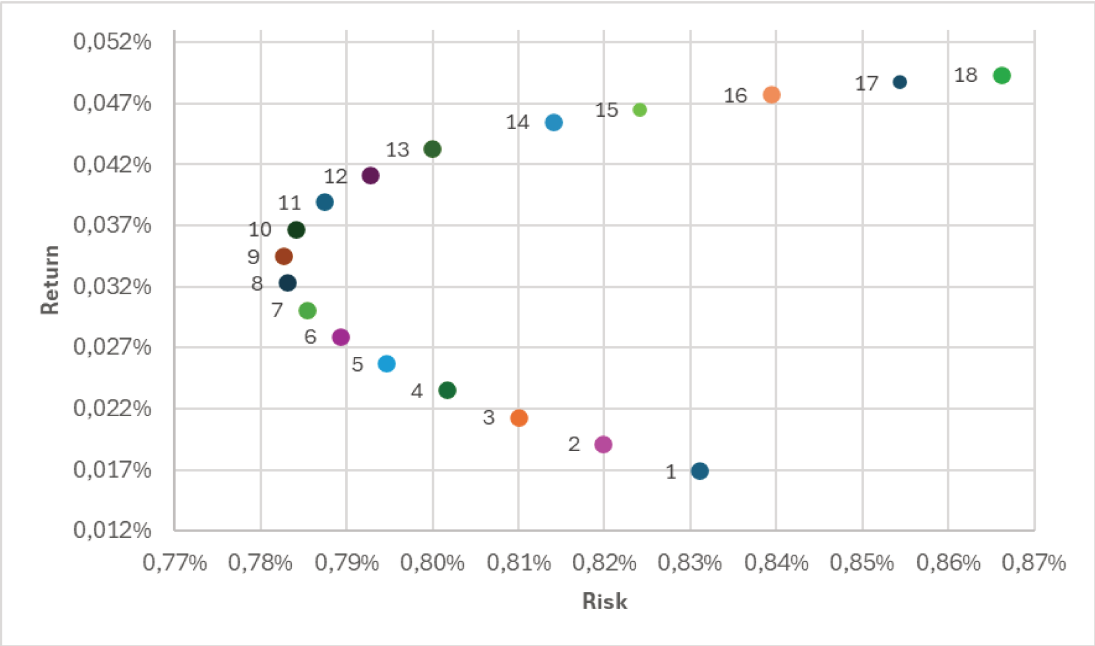
4.4 Efficient Frontier

In a new stage, an analysis of alternative portfolios to the previously described was carried out, therefore, different expected returns were established, and optimisations were made for each of these values using Solver.

For this optimization, variance minimisation was used as an objective, the cells that the system was allowed to adjust were the investment proportions in each index and the requested return was used as a constraint.

In this way, 18 portfolios with their respective risks and returns were obtained[12]. All possible combinations of risky assets are located along the efficient frontier.

Graph 5: Efficient frontier



Source: own creation based on data historical data and using Solver

In the above graph is possible to observe that for each portfolio located at the bottom of the curve (1 to 7) there is another option at the top of the curve which offers a higher return for the same level of risk (10 to 18). Therefore, the bottom

[12] Details of risk and return for each portfolio can be found on Appendix.

part of the curve shows inefficient portfolios, and only the top part should be considered, those are options with a better return-risk trade-off.

Being the third objective the achievement of three different portfolios according to alternative risk profiles, it is possible to establish them considering the already created portfolios.

The portfolio with the lowest risk is number 9 (already explained as minimum variance portfolio), and it can be considered as an ideal option for a risk averse investor, also portfolios 10 and 11 can be included because the risk does not increase much, and the return improve.

The composition in the three cases is more than 50% in S&P 500, then almost 30% in Nikkei 225 and the remaining percentage in FTSE 100, to achieve the lowest possible risk it does not include any proportion of the other indices.

For a risk neutral investor, portfolios 12, 13 and 14 can be considered, with risk from 0.79% to 0.81% and return rate from 0.041% to 0.045%.

For these options, the composition is once again similar, only in portfolio 12 FTSE 100 is included, but only a 5,12%, then S&P 500 66,34% and the remaining 28,54% in the Japanese index. For alternatives 13 and 14 the proposal is to only invest in the American and Japanese indices, with a bigger proportion in the first one.

Portfolios from 15 to 18 are good options for risk lovers, is easily to see how the return increases at the same time as risk does it; this kind of investors will demand a higher return in exchange for greater risk exposure.

While portfolio 18 is the one that only invest in S&P 500 the 100%, and offers the greater possible return (0.049%), the other three options propose a portfolio consisting of S&P 500 and Nikkei 225, as higher returns are demanded, the proportion to be invested in the first index increases and decreases in the second.

The results obtained are consistent with the correlation coefficients shown, as the S&P 500 and the Nikkei 225 have a low correlation and the proposed portfolios use them to diversify in order to reduce risk, and they also have the

best expected returns, which is also taken into account when constructing an efficient portfolio.

On the other hand, Latin American indices both have negative expected returns, and it makes sense not to include them in the portfolios proposed by the optimisation tool.

4.5 Capital Asset Pricing Model

Despite being the main subject of the present research equity portfolios, given that CAPM was explained in section 3, it will be analysed what happen when the risk-free asset is added.

As a risk-free asset, 10-year US Treasury bills were used and their daily yields for the period under analysis were considered. The average annual yield was calculated and then, using this data, the average for the 11-year period[13].

The daily[14] average rate obtained was 0.008% and will be used as the risk-free rate.

Considering only the efficient portfolios within the efficient frontier, it is possible to add the capital market line, which is a line that starts in the risk-free asset and is tangent to the market portfolio (graph 6).

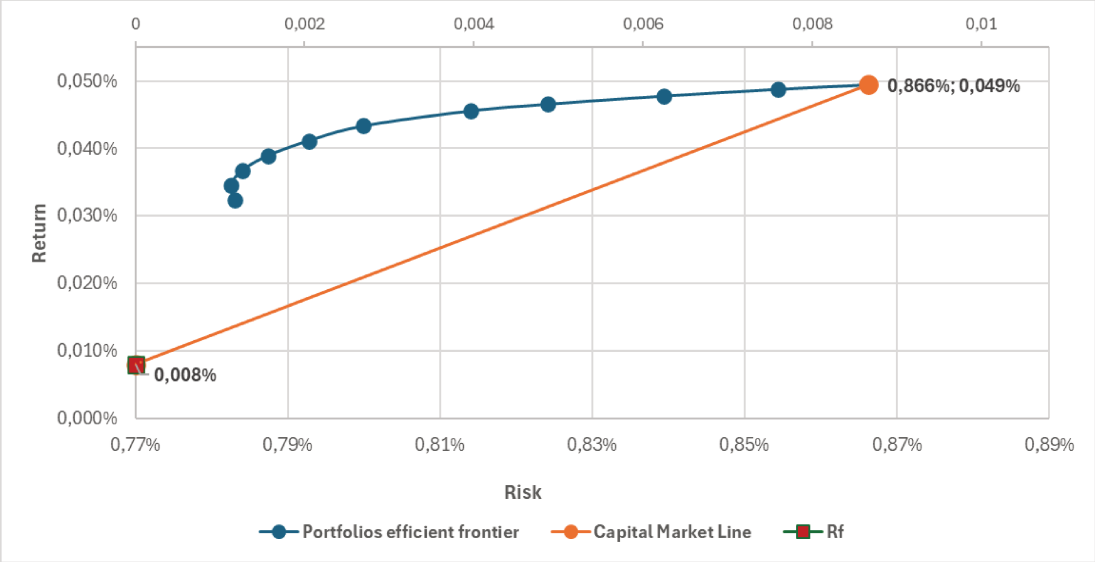
For this purpose, a portfolio that invests 100% in the risk-free asset was considered (red square on graph 6), it has a return of 0.008% and no risk.

On the other hand, the market portfolio is determined by the portfolio that include all the available assets and maximizes return, it can also be considered the one with the highest Sharpe ratio[15] and since the developed analysis is considering portfolios made of different market indices and not by individual assets, the optimal portfolio to obtain the best possible return has already been found and coincides with the portfolio of the S&P 500 index (Risk: 0.866%;

[13] Detailed average rate per year can be found in the appendix. Information was retrieved from https://fred.stlouisfed.org/
[14] 252 trading days were considered to calculate daily average from the annual return.
[15] Sharpe ratios for all the portfolios can be found on appendix B.

return: 0.049%; Sharpe ratio 0.0478). It should be considered that this is a widely diversify index, and it is the nearness to the market portfolio.

Graph 6: Efficient frontier and CML



Source: own creation based on data historical data and using Solver

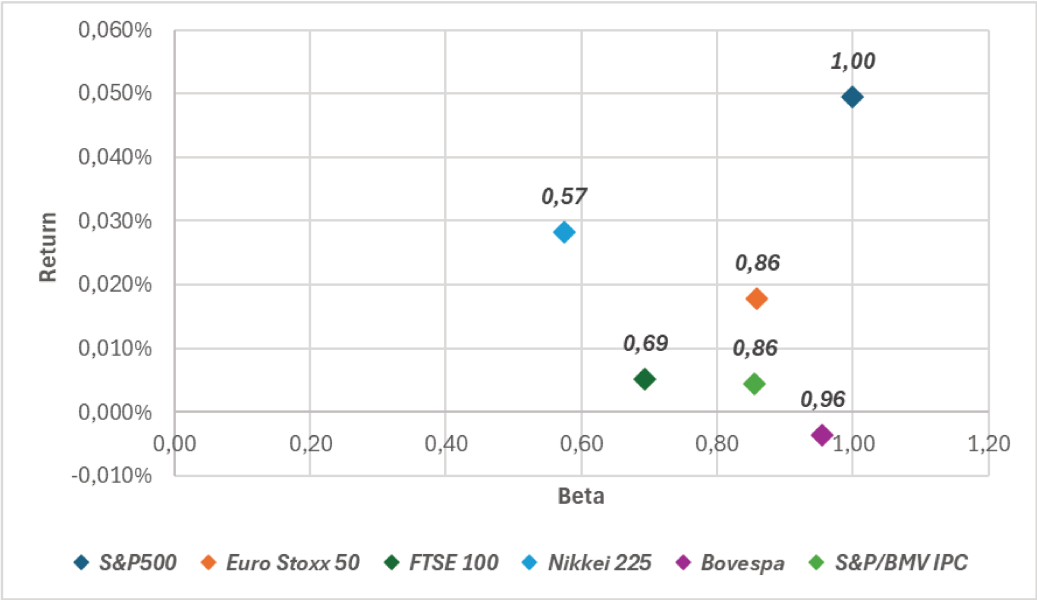
The CML is a special case of the CAL, it defines the market portfolio, which is the best achievable option for all the investors, whether the CAL offers different combinations of risk-free and risky assets, and it will depend on the preferences of each investor which one to choose, therefore, in which section of the line each of them will prefer to be placed.

If instead of risk, measured through standard deviation, only the measure of systematic risk through beta is considered on the X-axis, the security market line can be plotted. The SML graphs individual asset risk premiums.

In a first instance, betas calculated using S&P500 as reference index were calculated and are plotted on the following graph. The betas were obtained dividing the covariance of each index with S&P500 by the variance of this index (the reference index).

The Brazilian index has the closest beta to 1, followed by the European and Mexican indices, while FTSE 100 and Nikkei 225 have lower betas.

Graph 7: Betas

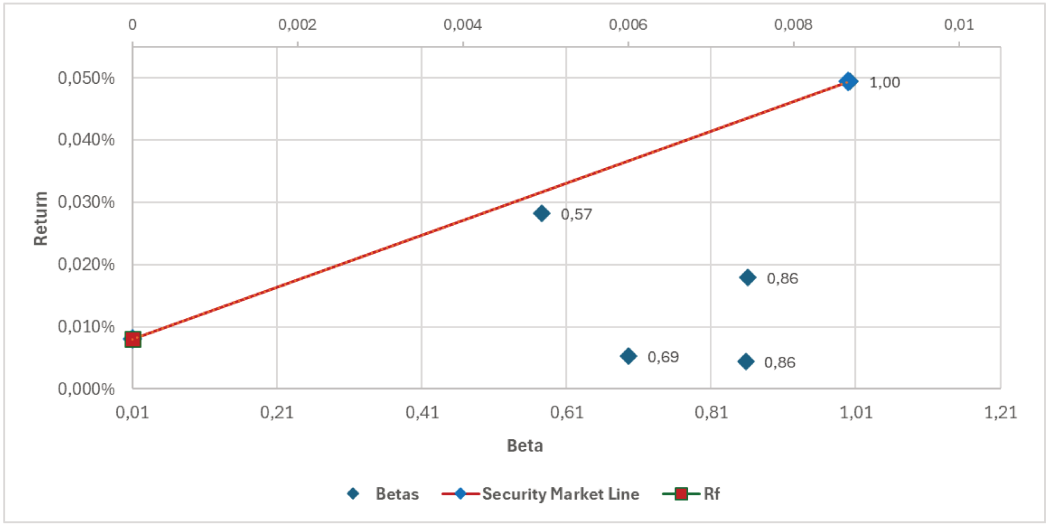


Source: own creation based on data historical data and using Solver

With betas being calculated, it is possible to draw the SML, that goes from the risk free to the market portfolio, which is shown is graph 8.

The objective of SML is to check whether the market is pricing assets correctly, through the location of them on the graph, being undervalued if they are above the SML and overvalued when they are below it.

Graph 8: Security Market Line



Source: own creation based on data historical data and using Solver

To apply IAPM it will be necessary to make some clarifications: since the whole analysis was based on market indices, S&P500 has been taken as the representative index for world market. It should be noted that while the S&P 500 is representative of developed markets, it may not fully reflect the geographic diversification of a truly global portfolio, yet for the purposes of the analysis it was used as a proxy.

Regarding the risk-free rate, 0,008% was used (calculation was already explained on section 4.5), since all the markets are globalized, this rate was considered both as domestic as well as international.

Then, regarding international beta, it should be taking into account that the beta for domestic market is 1 in all the cases, because it is the result of dividing covariance of each market on itself by the variance of each individual index and the beta for each market and S&P500 has already been calculated.

Therefore, applying the formula exposed in 3.3.1, it is possible to obtain the expected returns for each country, that are the following:

Table 5: Expected returns using IAPM

Expected return using formula for IAPM	
S&P500	0,0494%
Euro Stoxx 50	0,0436%
FTSE 100	0,0367%
Nikkei 225	0,0318%
Bovespa	0,0476%
S&P/BMV IPC	0,0435%

Source: own creation

Except for the American index, where the expected return calculated through IAPM and the one already presented using historical data coincide, for all the other indices, the expected return applying the international model shows a significant improvement.

The fact that the expected return matches can be attributed to the fact that the S&P500 is a proxy for the world market, and its international beta is equal to 1.

The IAPM model in the case of the S&P500 is adjusted so that its expected return is equal to the historical return due to the assumption of efficient markets and a globalised market with constant conditions.

For all the other cases, the expected returns differ from the historical ones, because the IAPM formula considers market factors such as risk-free rate and the market risk premium, basically these are notions of expected returns based on future market conditions.

In summary, the expected return is calculated as a function of international systematic risk and global market conditions and reflects the compensation investors should receive for taking risk of the asset, measured it through its international beta.

It allows investors to compare different assets or international markets in terms of it expected return, adjusted by global risk. Therefore, it is a useful tool for making investment decisions, especially when investing internationally, as well as to apply diversification strategies.

5. Conclusion

This research explored equity portfolio optimization through international diversification, aiming to determine whether diversification could increase portfolio efficiency

The analysis has led to different findings, which are presented below:

-Latin American indices are the most volatile and also have the lowest returns, in comparison to indices considered in this research. On the other hand, S&P 500 is the most stable and profitable of all indices. Although it has not been the scope of this research, these behaviours are linked to both global as well as local significant events, which could have been of economic, political or other nature.

Both Brazilian and Mexican markets are smaller in comparison to the others, and as it was mentioned in section three, emerging markets are more easily affected by restrictions, and also have bigger exchange rate variations which could affect foreign investors.

Once the individual characteristics of each index had been studied, the performance of the diversified portfolios was analysed, where the effects of the combination of indices became more evident.

-Given the risk-return trade-off of each index individually, the portfolio with equal proportions in all of them has a lower risk but also a lower return and with a slightly increment on risk better returns can be achieved, therefore it is not this portfolio an efficient option.

-If the variance is minimized, a portfolio composed of three indices is reached, which has more than 50% of S&P 500 and the rest divided between FTSE 100 and Nikkei 225. Those three indices are low correlated, which demonstrates the positive effects of diversification. Even though the expected return for this portfolio is lower than investing only in the American index, the risk the investor would assume is also smaller.

-The efficient frontier gave and overview of different portfolio combinations, where the European and Latin American indices were excluded, once again

the priority was given to the three already mentioned indices that the system took for the previous portfolios.

Brazilian and Mexican indices showed negative expected returns plus a high volatility, however, this is not the case for the European index, and its exclusion seems to be linked to the strong correlation it has with S&P 500 and FTSE 100, therefore, it does not offer any gain for diversification.

The alternative portfolios presented for different risk profiles were composed of different proportions of only three indices, with a clear predominance of S&P 500.

When drawing the CML, once again the portfolio that invest only in S&P 500 appears as the optimal one, becoming in this case the market portfolio.

It is important to emphasize that using the market indices that include a big portfolio of assets of each country may have biased the analysis, also the inclusion of six indices may have been excessive to analyse this topic.

Although the American index has dominated all analyses, given its strong performance, it would not have been possible not to consider it, since, as mentioned above, it is the largest market in the world and therefore, when seeking to diversify a portfolio, the option of investing in this country is always considered.

From a practical point of view, this research provides valuable insights for investors seeking to optimize their portfolios: non-US investors, can improve their risk-return trade-off by adding American assets to their portfolios, achieving better results than investing only in domestic markets. While for US-investors international diversification will not lead to a higher return, but it can be used to reduce the risk for example in the case of a risk-averse investor.

These results demonstrate that international diversification can increase portfolio efficiency, offering substantial benefits for risk management and return optimization.

The models presented and applied to the analysis are valuable tools for making investment decisions as well as to make predictions when evaluating alternative portfolios or creating diversification strategies.

It should be highlighted that different instruments are available to invest in foreign markets, as well as hedging tools to prevent undesired effects in the portfolio performance, especially to mitigate the risks associated with currency fluctuations.

Despite the home bias effect, international diversification offers valuable opportunities for investors in emerging countries, allowing them to achieve better returns and reduce the risk exposure at the same time.

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Appendixes

Appendix A

Current portfolios composition.

- ***Bovespa:***

IBOV - Carteira Set. a Dez. 2024[16]			
Código	***Ação***	***Qtde. Teórica***	***Part. (%)***
ABEV3	AMBEV S/A	4.394.835.131	2,517
ALOS3	ALLOS	502.481.592	0,51
ALPA4	ALPARGATAS	166.460.180	0,057
ASAI3	ASSAI	1.349.687.675	0,575
AURE3	AUREN	291.727.616	0,148
AZUL4	AZUL	328.421.113	0,079
AZZA3	AZZAS 2154	127.330.493	0,278
B3SA3	B3	5.511.401.013	3,113
BBAS3	BRASIL	2.842.613.858	3,56
BBDC3	BRADESCO	1.484.426.957	0,935
BBDC4	BRADESCO	5.129.958.973	3,574
BBSE3	BBSEGURIDADE	637.332.335	1,043
BEEF3	MINERVA	261.359.935	0,087
BPAC11	BTGP BANCO	1.287.247.964	2,039
BRAP4	BRADESPAR	250.969.312	0,218
BRFS3	BRF SA	814.523.002	0,952
BRKM5	BRASKEM	265.388.400	0,213
CCRO3	CCR SA	991.920.937	0,585
CMIG4	CEMIG	1.858.636.840	0,963
CMIN3	CSNMINERACAO	1.110.559.345	0,281
COGN3	COGNA ON	1.872.454.628	0,114
CPFE3	CPFL ENERGIA	187.732.538	0,285
CPLE6	COPEL	1.679.233.590	0,78
CRFB3	CARREFOUR BR	531.901.983	0,213
CSAN3	COSAN	1.165.337.843	0,693
CSNA3	SID NACIONAL	736.268.400	0,389
CVCB3	CVC BRASIL	525.591.097	0,046
CXSE3	CAIXA SEGURI	517.500.000	0,377
CYRE3	CYRELA REALT	264.710.610	0,257

[16] Source: https://www.b3.com.br/pt_br/market-data-e-indices/indices/indices-amplos/indice-ibovespa-ibovespa-composicao-da-carteira.htm

EGIE3	ENGIE BRASIL	255.236.938	0,513
ELET3	ELETROBRAS	1.780.013.881	3,31
ELET6	ELETROBRAS	268.733.136	0,552
EMBR3	EMBRAER	734.632.601	1,532
ENEV3	ENEVA	1.579.821.370	0,924
ENGI11	ENERGISA	326.175.300	0,687
EQTL3	EQUATORIAL	1.244.602.400	1,896
EZTC3	EZTEC	97.334.950	0,061
FLRY3	FLEURY	464.420.650	0,33
GGBR4	GERDAU	1.242.683.687	1,013
GOAU4	GERDAU MET	661.577.619	0,309
HAPV3	HAPVIDA	4.779.395.040	0,903
HYPE3	HYPERA	409.310.634	0,52
IGTI11	IGUATEMI S.A	211.468.849	0,208
IRBR3	IRBBRASIL RE	81.838.243	0,176
ITSA4	ITAUSA	5.504.638.590	2,682
ITUB4	ITAUUNIBANCO	4.455.161.368	7,28
JBSS3	JBS	1.142.696.472	1,781
KLBN11	KLABIN S/A	765.785.673	0,736
LREN3	LOJAS RENNER	952.487.418	0,722
LWSA3	LWSA	420.626.825	0,085
MGLU3	MAGAZ LUIZA	353.448.195	0,191
MRFG3	MARFRIG	302.019.876	0,196
MRVE3	MRV	375.335.828	0,125
MULT3	MULTIPLAN	318.010.631	0,363
NTCO3	GRUPO NATURA	849.350.756	0,513
PCAR3	P.ACUCAR-CBD	379.327.171	0,053
PETR3	PETROBRAS	2.448.348.744	4,683
PETR4	PETROBRAS	4.431.132.660	7,77
PETZ3	PETZ	307.729.428	0,067
PRIO3	PETRORIO	798.909.771	1,667
RADL3	RAIADROGASIL	1.279.770.315	1,571
RAIL3	RUMO S.A.	1.215.994.115	1,183
RAIZ4	RAIZEN	1.192.947.233	0,171
RDOR3	REDE D OR	1.145.289.019	1,629
RECV3	PETRORECSA	275.699.226	0,251
RENT3	LOCALIZA	977.700.485	1,802
RRRP3	3R PETROLEUM	463.981.130	0,545
SANB11	SANTANDER BR	356.586.730	0,499
SBSP3	SABESP	683.508.570	2,884
SLCE3	SLC AGRICOLA	194.261.422	0,153
SMTO3	SAO MARTINHO	142.348.116	0,176
STBP3	SANTOS BRP	855.712.622	0,495

SUZB3	SUZANO S.A.	630.821.784	1,545
TAEE11	TAESA	218.568.234	0,345
TIMS3	TIM	807.495.418	0,642
TOTS3	TOTVS	540.206.440	0,721
TRPL4	TRAN PAULIST	395.801.044	0,436
UGPA3	ULTRAPAR	1.090.134.379	1,134
USIM5	USIMINAS	515.193.199	0,143
VALE3	VALE	4.270.903.023	11,334
VAMO3	VAMOS	485.166.826	0,164
VBBR3	VIBRA	1.023.392.529	1,172
VIVA3	VIVARA S.A.	125.823.025	0,148
VIVT3	TELEF BRASIL	407.257.128	0,94
WEGE3	WEG	1.269.032.828	3,061
YDUQ3	YDUQS PART	290.010.465	0,127
Quantidade Teórica Total		*95.883.873.369*	*100*

- ***S&P/BMV IPC***[17]

COMPONENTE	***TICKER***
Alfa SA A	ALFA A
Alsea SA	ALSEA *
America Movil SAB de CV B	AMX B
Arca Continental, SAB de CV	AC *
Banco del Bajio, S.A.	BBAJIO O
Becle, S.A. De C.V.	CUERVO *
Bolsa Mexicana de Valores SA de CV	BOLSA A
Cemex SA CPO	CEMEX CPO
Coca-Cola Femsa SAB de CV UBL	KOF UBL
Corporacion Inmobiliaria Vesta, S.A.B. DE C.V.	VESTA *
El Puerto de Liverpool SAB de CV	LIVEPOL C-1
Fomento Economico Mexicano S.A.B. de C.V.	FEMSA UBD
Genomma Lab Internacional SA de CV	LAB B
Gentera SAB de CV	GENTERA *
Gruma SAB B	GRUMA B

[17] Retrieved from https://www.spglobal.com/spdji/es/indices/equity/sp-bmv-ipc/#data, information from 31st October 2024

Grupo Aeroportuario del Centro Norte, S.A.B. de C.V.	OMA B
Grupo Aeroportuario del Pacifico, S.A.B. de C.V.	GAP B
Grupo Aeroportuario del Sureste SAB de CV B	ASUR B
Grupo Bimbo S.A.B.	BIMBO A
Grupo Carso SAB de CV	GCARSO A1
Grupo Cementos de Chihuahua SAB de CV	GCC *
Grupo Comercial Chedraui SA de CV	CHDRAUI B
Grupo Financiero Banorte O	GFNORTE O
Grupo Financiero Inbursa O	GFINBUR O
Grupo Mexico SAB de CV B	GMEXICO B
Grupo Televisa SAB CPO	TLEVISA CPO
Industrias Penoles	PE&OLES *
Kimberly Clark de Mexico S.A.B. de C.V. A	KIMBER A
La Comer S.A.B. de C.V. UBC	LACOMER UBC
Megacable Holdings SAB de CV	MEGA CPO
ORBIA ADVANCE CORPORATION SAB de CV	ORBIA *
Promotora y Operadora de Infraestructura SAB de CV	PINFRA *
Qualitas Controladora S.A.B de C.V.	Q *
Regional, S.A. de C.V.	R A
Walmart de Mexico SAB de CV	WALMEX *

- ***FTSE 100[18]***

Code	***Name***	***Market cap (m)***
LAND	LAND SECURITIES GROUP PLC ORD 10 2/3P	4,506.87
EZJ	EASYJET PLC ORD 27 2/7P	4,069.00
FRES	FRESNILLO PLC ORD USD0.50	4,775.07
LLOY	LLOYDS BANKING GROUP PLC ORD 10P	34,261.50
BATS	BRITISH AMERICAN TOBACCO PLC ORD 25P	63,408.72
SBRY	SAINSBURY(J) PLC ORD 28 4/7P	5,765.89
FRAS	FRASERS GROUP PLC ORD 10P	3,368.36
AV.	AVIVA PLC ORD 32 17/19P	12,981.26
PRU	PRUDENTIAL PLC ORD 5P	17,147.87
AAL	ANGLO AMERICAN PLC ORD USD0.54945	30,503.46
MNG	M&G PLC ORD £0.05	4,731.22

[18] Retrieved from https://www.londonstockexchange.com/indices/ftse-100/constituents/table

HSBA	HSBC HLDGS PLC ORD $0.50 (UK REG)	129,673.02
ANTO	ANTOFAGASTA PLC ORD 5P	16,365.22
RIO	RIO TINTO PLC ORD 10P	60,192.54
LGEN	LEGAL & GENERAL GROUP PLC ORD 2 1/2P	12,905.43
BT.A	BT GROUP PLC ORD 5P	14,136.82
PHNX	PHOENIX GROUP HOLDINGS PLC ORD 10P	5,023.05
GLEN	GLENCORE PLC ORD USD0.01	46,283.40
HIK	HIKMA PHARMACEUTICALS PLC ORD SHS 10P	4,029.46
JD.	JD SPORTS FASHION PLC ORD 0.05P	6,136.83
SMDS	SMITH (DS) PLC ORD 10P	7,786.03
SHEL	SHELL PLC ORD EUR0.07	158,091.30
AAF	AIRTEL AFRICA PLC ORD USD0.50	3,538.05
HLN	HALEON PLC ORD GBP0.01	33,144.35
IAG	INTL CONSOLIDATED AIRLINES GROUP SA ORD EUR0.10 (CDI)	11,845.87
BLND	BRITISH LAND CO PLC ORD 25P	3,798.23
TSCO	TESCO PLC ORD 6 1/3P	23,356.77
BARC	BARCLAYS PLC ORD 25P	37,515.74
KGF	KINGFISHER PLC ORD 15 5/7P	5,309.84
EDV	ENDEAVOUR MINING PLC ORD USD0.01	3,733.06
BP.	BP PLC $0.25	61,497.82
IMB	IMPERIAL BRANDS PLC ORD 10P	19,915.32
RKT	RECKITT BENCKISER GROUP PLC ORD 10P	32,829.78
UTG	UNITE GROUP PLC ORD 25P	4,188.95
NWG	NATWEST GROUP PLC ORD 107.69P	31,618.91
CTEC	CONVATEC GROUP PLC ORD 10P	5,099.88
STAN	STANDARD CHARTERED PLC ORD USD0.50	23,044.02
SVT	SEVERN TRENT PLC ORD 97 17/19P	8,075.32
ULVR	UNILEVER PLC ORD 3 1/9P	112,548.35
SSE	SSE PLC ORD 50P	18,929.48
VOD	VODAFONE GROUP PLC ORD USD0.20 20/21	18,058.91
SN.	SMITH & NEPHEW PLC ORD USD0.20	8,428.97
MKS	MARKS & SPENCER GROUP PLC ORD 1P	7,552.20
SMIN	SMITHS GROUP PLC ORD 37.5P	5,847.25
MNDI	MONDI PLC ORD EUR 0.22	5,149.08
NG.	NATIONAL GRID PLC ORD 12 204/473P	47,629.18
PSN	PERSIMMON PLC ORD 10P	4,069.24
WEIR	WEIR GROUP PLC ORD 12.5P	5,550.51
UU.	UNITED UTILITIES GROUP PLC ORD 5P	7,398.49
IHG	INTERCONTINENTAL HOTELS GROUP PLC ORD 20 340/399P	15,080.57
RTO	RENTOKIL INITIAL PLC ORD 1P	10,118.36
BKG	BERKELEY GROUP HOLDINGS (THE) PLC ORD 5.6110477936P	4,395.31
SDR	SCHRODERS PLC ORD 20P	5,041.00

LSEG	LONDON STOCK EXCHANGE GROUP PLC ORD SHS 6 79/86P	56,544.19
AUTO	AUTO TRADER GROUP PLC ORD 1P	7,105.15
ADM	ADMIRAL GROUP PLC ORD 0.1P	7,725.00
HL.	HARGREAVES LANSDOWN PLC ORD 0.4P	5,174.82
NXT	NEXT PLC ORD 10P	11,890.31
BTRW	BARRATT REDROW PLC ORD 10P	5,941.59
HWDN	HOWDEN JOINERY GROUP PLC ORD 10P	4,521.67
HSX	HISCOX LD ORD 6.5P (DI)	3,521.11
PSON	PEARSON PLC ORD 25P	7,934.73
TW.	TAYLOR WIMPEY PLC ORD 1P	4,658.57
BNZL	BUNZL PLC ORD 32 1/7P	11,432.08
DCC	DCC PLC ORD EUR0.25 (CDI)	5,339.32
LMP	LONDONMETRIC PROPERTY PLC ORD 10P	3,848.94
CNA	CENTRICA PLC ORD 6 14/81P	6,209.95
IMI	IMI PLC ORD 28 4/7P	4,391.68
CCH	COCA-COLA HBC AG ORD CHF6.70 (CDI)	9,970.74
CPG	COMPASS GROUP PLC ORD 11 1/20P	44,268.74
III	3I GROUP PLC ORD 73 19/22P	33,348.30
RMV	RIGHTMOVE PLC ORD 0.1P	4,750.66
BEZ	BEAZLEY PLC ORD 5P	4,708.65
SGRO	SEGRO PLC ORD 10P	10,330.43
DGE	DIAGEO PLC ORD 28 101/108P	52,167.95
AHT	ASHTEAD GROUP PLC ORD 10P	27,042.56
FCIT	F&C INVESTMENT TRUST PLC ORD 25P	5,339.59
ENT	ENTAIN PLC ORD EUR0.01	4,735.96
INF	INFORMA PLC ORD 0.1P	11,224.61
WTB	WHITBREAD PLC ORD 76 122/153P	5,126.03
RR.	ROLLS-ROYCE HOLDINGS PLC ORD SHS 20P	45,824.38
SPX	SPIRAX GROUP PLC ORD 26 12/13P	4,851.04
SGE	SAGE GROUP PLC ORD 1 4/77P	10,696.58
PSH	PERSHING SQUARE HOLDINGS LTD ORD NPV	6,546.80
EXPN	EXPERIAN PLC ORD USD0.10	33,155.92
ITRK	INTERTEK GROUP PLC ORD 1P	7,133.58
WPP	WPP PLC ORD 10P	8,878.66
SMT	SCOTTISH MORTGAGE INV TST PLC ORD 5P	11,662.81
BA.	BAE SYSTEMS PLC ORD 2.5P	38,869.50
HLMA	HALMA PLC ORD 10P	9,506.32
ICG	INTERMEDIATE CAPITAL GROUP PLC ORD 26 1/4P	5,806.83
VTY	VISTRY GROUP PLC ORD 50P	2,323.25
DPLM	DIPLOMA PLC ORD 5P	6,023.41
ABF	ASSOCIATED BRITISH FOODS PLC ORD 5 15/22P	16,159.18
MRO	MELROSE INDUSTRIES PLC ORD GBP0.001	6,302.81
REL	RELX PLC ORD 14 51/116P	65,748.77

AZN	ASTRAZENECA PLC ORD SHS $0.25	154,689.32
CRDA	CRODA INTERNATIONAL PLC ORD 10.609756P	4,877.47
GSK	GSK PLC ORD 31 1/4P	53,927.75
BME	B&M EUROPEAN VALUE RETAIL S.A. ORD 10P (DI)	3,798.32

- ***S&P500***

Top 10 constituents by weighting in the index:

Component	***Symbol***	***Sector****
Apple Inc.	*AAPL*	Information Technology
Nvidia Corp	*NVDA*	Information Technology
Microsoft Corp	*MSFT*	Information Technology
Consumer Discretionary	*AMZN*	Consumer Discretionary
Meta Platforms, Inc. Class A	*META*	Communication Services
Alphabet Inc A	*GOOGL*	Communication Services
Alphabet Inc C	*GOOG*	Communication Services
Berkshire Hathaway B	*BRK.B*	Financials
Broadcom Inc	*AVGO*	Information Technology
Tesla, Inc	*TSLA*	Consumer Discretionary

Source: https://www.spglobal.com/spdji/es/indices/equity/sp-500/#data

Breakdown by Sector:

Sector	**Ponderation**
Information Technology	31,70%
Financials	13,40%
Health Care	11,20%
Consumer Discretionary	10,00%
Communication Services	9,10%
Industrials	8,50%
Consumer Staples	5,80%
Energy	3,40%
Utilities	2,50%
Real Estate	2,30%
Materials	2,20%

Source: https://www.spglobal.com/spdji/es/indices/equity/sp-500/#data

- **Euro Stoxx 50**

Top 10 components[19]:

Company	***Supersector***	***Country***	***Weight***
ASML HLDG	Technology	Netherlands	7,32%
SAP	Technology	Germany	6,46%
LVMH MOET HENNESSY	Consumer Products and Services	France	4,60%
TOTALENERGIES	Energy	France	4,06%
SCHNEIDER ELECTRIC	Industrial Goods and Services	France	4,03%
SIEMENS	Industrial Goods and Services	Germany	3,96%
ALLIANZ	Insurance	Germany	3,34%
SANOFI	Health Care	France	3,29%
DEUTSCHE TELEKOM	Telecommunications	Germany	2,94%
AIR LIQUIDE	Chemicals	France	2,81%

- **Nikkei 225[20]**

Company Name	***Sector***
KYOWA KIRIN CO., LTD.	Pharmaceuticals
TAKEDA PHARMACEUTICAL CO., LTD.	Pharmaceuticals
ASTELLAS PHARMA INC.	Pharmaceuticals
SUMITOMO PHARMA CO., LTD.	Pharmaceuticals
SHIONOGI & CO., LTD.	Pharmaceuticals
CHUGAI PHARMACEUTICAL CO., LTD.	Pharmaceuticals
EISAI CO., LTD.	Pharmaceuticals
DAIICHI SANKYO CO., LTD.	Pharmaceuticals
OTSUKA HOLDINGS CO., LTD.	Pharmaceuticals
MINEBEA MITSUMI INC.	Electric Machinery
HITACHI, LTD.	Electric Machinery
MITSUBISHI ELECTRIC CORP.	Electric Machinery
FUJI ELECTRIC CO., LTD.	Electric Machinery
YASKAWA ELECTRIC CORP.	Electric Machinery
SOCIONEXT INC.	Electric Machinery
NIDEC CORP.	Electric Machinery
OMRON CORP.	Electric Machinery
GS YUASA CORP.	Electric Machinery
NEC CORP.	Electric Machinery
FUJITSU LTD.	Electric Machinery

[19] Retrieved from https://stoxx.com/index/sx5e/?factsheet=true
[20] Retrieved from https://indexes.nikkei.co.jp/en/nkave/index/component?idx=nk225

RENESAS ELECTRONICS CORP.	Electric Machinery
SEIKO EPSON CORP.	Electric Machinery
PANASONIC HOLDINGS CORP.	Electric Machinery
SHARP CORP.	Electric Machinery
SONY GROUP CORP.	Electric Machinery
TDK CORP.	Electric Machinery
ALPS ALPINE CO., LTD.	Electric Machinery
YOKOGAWA ELECTRIC CORP.	Electric Machinery
ADVANTEST CORP.	Electric Machinery
KEYENCE CORP.	Electric Machinery
DENSO CORP.	Electric Machinery
LASERTEC CORP.	Electric Machinery
CASIO COMPUTER CO., LTD.	Electric Machinery
FANUC CORP.	Electric Machinery
KYOCERA CORP.	Electric Machinery
TAIYO YUDEN CO., LTD.	Electric Machinery
MURATA MANUFACTURING CO., LTD.	Electric Machinery
SCREEN HOLDINGS CO., LTD.	Electric Machinery
CANON INC.	Electric Machinery
RICOH CO., LTD.	Electric Machinery
TOKYO ELECTRON LTD.	Electric Machinery
NISSAN MOTOR CO., LTD.	Automobiles & Auto parts
ISUZU MOTORS LTD.	Automobiles & Auto parts
TOYOTA MOTOR CORP.	Automobiles & Auto parts
HINO MOTORS, LTD.	Automobiles & Auto parts
MITSUBISHI MOTORS CORP.	Automobiles & Auto parts
MAZDA MOTOR CORP.	Automobiles & Auto parts
HONDA MOTOR CO., LTD.	Automobiles & Auto parts
SUZUKI MOTOR CORP.	Automobiles & Auto parts
SUBARU CORP.	Automobiles & Auto parts
YAMAHA MOTOR CO., LTD.	Automobiles & Auto parts
TERUMO CORP.	Precision Instruments
KONICA MINOLTA, INC.	Precision Instruments
DISCO CORP.	Precision Instruments
NIKON CORP.	Precision Instruments
OLYMPUS CORP.	Precision Instruments
HOYA CORP.	Precision Instruments
CITIZEN WATCH CO., LTD.	Precision Instruments
NIPPON TELEGRAPH & TELEPHONE CORP.	Communications
KDDI CORP.	Communications
SOFTBANK CORP.	Communications
NTT DATA GROUP CORP.	Communications
SOFTBANK GROUP CORP.	Communications

SHIZUOKA FINANCIAL GROUP, INC.	Banking
CONCORDIA FINANCIAL GROUP, LTD.	Banking
AOZORA BANK, LTD.	Banking
MITSUBISHI UFJ FINANCIAL GROUP, INC.	Banking
RESONA HOLDINGS, INC.	Banking
SUMITOMO MITSUI TRUST GROUP, INC.	Banking
SUMITOMO MITSUI FINANCIAL GROUP, INC.	Banking
THE CHIBA BANK, LTD.	Banking
FUKUOKA FINANCIAL GROUP, INC.	Banking
MIZUHO FINANCIAL GROUP, INC.	Banking
CREDIT SAISON CO., LTD.	Other Financial Services
ORIX CORP.	Other Financial Services
JAPAN EXCHANGE GROUP, INC.	Other Financial Services
DAIWA SECURITIES GROUP INC.	Securities
NOMURA HOLDINGS, INC.	Securities
SOMPO HOLDINGS, INC.	Insurance
MS&AD INSURANCE GROUP HOLDINGS, INC.	Insurance
DAI-ICHI LIFE HOLDINGS, INC.	Insurance
TOKIO MARINE HOLDINGS, INC.	Insurance
T&D HOLDINGS, INC.	Insurance
NISSUI CORP.	Fishery
NISSHIN SEIFUN GROUP INC.	Foods
MEIJI HOLDINGS CO., LTD.	Foods
NH FOODS LTD.	Foods
SAPPORO HOLDINGS LTD.	Foods
ASAHI GROUP HOLDINGS, LTD.	Foods
KIRIN HOLDINGS CO., LTD.	Foods
KIKKOMAN CORP.	Foods
AJINOMOTO CO., INC.	Foods
NICHIREI CORP.	Foods
JAPAN TOBACCO INC.	Foods
J.FRONT RETAILING CO., LTD.	Retail
ZOZO, INC.	Retail
ISETAN MITSUKOSHI HOLDINGS LTD.	Retail
SEVEN & I HOLDINGS CO., LTD.	Retail
RYOHIN KEIKAKU CO., LTD.	Retail
TAKASHIMAYA CO., LTD.	Retail
MARUI GROUP CO., LTD.	Retail
AEON CO., LTD.	Retail
NITORI HOLDINGS CO., LTD.	Retail
FAST RETAILING CO., LTD.	Retail
M3, INC.	Services
DENA CO., LTD.	Services

NEXON CO., LTD.	Services
NOMURA RESEARCH INSTITUTE, LTD.	Services
DENTSU GROUP INC.	Services
MERCARI, INC.	Services
ORIENTAL LAND CO., LTD.	Services
LY CORP.	Services
TREND MICRO INC.	Services
CYBERAGENT, INC.	Services
RAKUTEN GROUP, INC.	Services
RECRUIT HOLDINGS CO., LTD.	Services
JAPAN POST HOLDINGS CO., LTD.	Services
NINTENDO CO., LTD.	Services
TOHO CO., LTD	Services
SECOM CO., LTD.	Services
KONAMI GROUP CORP.	Services
INPEX CORP.	Mining
TEIJIN LTD.	Textiles & Apparel
TORAY INDUSTRIES, INC.	Textiles & Apparel
OJI HOLDINGS CORP.	Pulp & Paper
KURARAY CO., LTD.	Chemicals
ASAHI KASEI CORP.	Chemicals
RESONAC HOLDINGS CORP.	Chemicals
SUMITOMO CHEMICAL CO., LTD.	Chemicals
NISSAN CHEMICAL CORP.	Chemicals
TOSOH CORP.	Chemicals
TOKUYAMA CORP.	Chemicals
DENKA CO., LTD.	Chemicals
SHIN-ETSU CHEMICAL CO., LTD.	Chemicals
MITSUI CHEMICALS, INC.	Chemicals
MITSUBISHI CHEMICAL GROUP CORP.	Chemicals
UBE CORP.	Chemicals
KAO CORP.	Chemicals
FUJIFILM HOLDINGS CORP.	Chemicals
SHISEIDO CO., LTD.	Chemicals
NITTO DENKO CORP.	Chemicals
IDEMITSU KOSAN CO., LTD.	Petroleum
ENEOS HOLDINGS, INC.	Petroleum
THE YOKOHAMA RUBBER CO., LTD.	Rubber
BRIDGESTONE CORP.	Rubber
AGC INC.	Glass & Ceramics
NIPPON ELECTRIC GLASS CO., LTD.	Glass & Ceramics
TAIHEIYO CEMENT CORP.	Glass & Ceramics
TOKAI CARBON CO., LTD.	Glass & Ceramics

TOTO LTD.	Glass & Ceramics
NGK INSULATORS, LTD.	Glass & Ceramics
NIPPON STEEL CORP.	Steel
KOBE STEEL, LTD.	Steel
JFE HOLDINGS, INC.	Steel
SUMCO CORP.	Nonferrous Metals
MITSUI MINING & SMELTING CO.	Nonferrous Metals
MITSUBISHI MATERIALS CORP.	Nonferrous Metals
SUMITOMO METAL MINING CO., LTD.	Nonferrous Metals
DOWA HOLDINGS CO., LTD.	Nonferrous Metals
FURUKAWA ELECTRIC CO., LTD.	Nonferrous Metals
SUMITOMO ELECTRIC IND., LTD.	Nonferrous Metals
FUJIKURA LTD.	Nonferrous Metals
SOJITZ CORP.	Trading Companies
ITOCHU CORP.	Trading Companies
MARUBENI CORP.	Trading Companies
TOYOTA TSUSHO CORP.	Trading Companies
MITSUI & CO., LTD.	Trading Companies
SUMITOMO CORP.	Trading Companies
MITSUBISHI CORP.	Trading Companies
COMSYS HOLDINGS CORP.	Construction companies
TAISEI CORP.	Construction companies
OBAYASHI CORP.	Construction companies
SHIMIZU CORP.	Construction companies
HASEKO CORP.	Construction companies
KAJIMA CORP.	Construction companies
DAIWA HOUSE IND. CO., LTD.	Construction companies
SEKISUI HOUSE, LTD.	Construction companies
JGC HOLDINGS CORP.	Construction companies
THE JAPAN STEEL WORKS, LTD.	Machinery
OKUMA CORP.	Machinery
AMADA CO., LTD.	Machinery
SMC CORP.	Machinery
KOMATSU LTD.	Machinery
SUMITOMO HEAVY IND., LTD.	Machinery
HITACHI CONST. MACH. CO., LTD.	Machinery
KUBOTA CORP.	Machinery
EBARA CORP.	Machinery
DAIKIN INDUSTRIES, LTD.	Machinery
NSK LTD.	Machinery
NTN CORP.	Machinery
JTEKT CORP.	Machinery
KANADEVIA CORP.	Machinery

MITSUBISHI HEAVY IND., LTD.	Machinery
IHI CORP.	Machinery
KAWASAKI HEAVY IND., LTD.	Shipbuilding
BANDAI NAMCO HOLDINGS INC.	Other Manufacturing
TOPPAN HOLDINGS INC.	Other Manufacturing
DAI NIPPON PRINTING CO., LTD.	Other Manufacturing
YAMAHA CORP.	Other Manufacturing
TOKYU FUDOSAN HOLDINGS CORP.	Real Estate
MITSUI FUDOSAN CO., LTD.	Real Estate
MITSUBISHI ESTATE CO., LTD.	Real Estate
TOKYO TATEMONO CO., LTD.	Real Estate
SUMITOMO REALTY & DEVELOPMENT CO., LTD.	Real Estate
TOBU RAILWAY CO., LTD.	Railway & Bus
TOKYU CORP.	Railway & Bus
ODAKYU ELECTRIC RAILWAY CO., LTD.	Railway & Bus
KEIO CORP.	Railway & Bus
KEISEI ELECTRIC RAILWAY CO., LTD.	Railway & Bus
EAST JAPAN RAILWAY CO.	Railway & Bus
WEST JAPAN RAILWAY CO.	Railway & Bus
CENTRAL JAPAN RAILWAY CO., LTD.	Railway & Bus
YAMATO HOLDINGS CO., LTD.	Land Transport
NIPPON EXPRESS HOLDINGS, INC.	Land Transport
NIPPON YUSEN K.K.	Marine Transport
MITSUI O.S.K.LINES, LTD.	Marine Transport
KAWASAKI KISEN KAISHA, LTD.	Marine Transport
JAPAN AIRLINES CO., LTD.	Air Transport
ANA HOLDINGS INC.	Air Transport
MITSUBISHI LOGISTICS CORP.	Warehousing
TOKYO ELECTRIC POWER COMPANY HOLDINGS, I	Electric Power
CHUBU ELECTRIC POWER CO., INC.	Electric Power
THE KANSAI ELECTRIC POWER CO., INC.	Electric Power
TOKYO GAS CO., LTD.	Gas
OSAKA GAS CO., LTD.	Gas

Appendix B

- Detailed data presented on Graph 1:

Annual yields						
Year	S&P 500	Euro stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
2012	10,42%	13,75%	7,86%	8,36%	-8,42%	22,53%
2013	25,86%	21,24%	13,82%	27,52%	-29,09%	-5,84%
2014	13,93%	-8,25%	-7,28%	-4,52%	-12,41%	-8,46%
2015	1,61%	-2,06%	-5,92%	9,32%	-37,47%	-10,88%
2016	11,06%	-1,93%	-4,15%	5,77%	71,88%	-10,48%
2017	18,56%	21,62%	15,95%	22,04%	20,59%	13,58%
2018	-8,43%	-20,47%	-18,83%	-12,63%	-7,70%	-18,19%
2019	27,06%	21,07%	14,54%	18,03%	16,81%	5,80%
2020	15,54%	4,44%	-10,50%	23,72%	-20,66%	-4,66%
2021	28,23%	11,62%	12,69%	-5,82%	-18,67%	15,86%
2022	-20,25%	-17,29%	-10,17%	-22,69%	13,95%	-1,34%
Average	**11,24%**	**3,98%**	**0,73%**	**6,28%**	**-1,02%**	**-0,19%**

- Detailed data presented on Graph 2:

	S&P 500	Euro stoxx 50	FTSE 100	Nikkei 225	Bovespa	S&P/BMV IPC
Average return (expected return)	0,049%	0,018%	0,005%	0,028%	-0,004%	0,004%
Standard deviation	0,866%	1,113%	0,955%	1,000%	1,710%	1,209%

- Alternative Portfolios: risk calculated using Solver for a given expected return.

Portfolios	Risk	Expected Return
1	0,8311%	0,0169%
2	0,8198%	0,0191%
3	0,8100%	0,0213%
4	0,8016%	0,0235%
5	0,7947%	0,0257%
6	0,7892%	0,0279%
7	0,7854%	0,0301%
8	0,7831%	0,0323%
9	0,7826%	0,0345%

10	0,7840%	0,0367%
11	0,7874%	0,0389%
12	0,7927%	0,0411%
13	0,7999%	0,0433%
14	0,8140%	0,0455%
15	0,8242%	0,0465%
16	0,8393%	0,0477%
17	0,8543%	0,0487%
18	0,8661%	0,0494%

- 10-years US Treasury bills – yield rates

Average yield	
Year	**%**
2012	1,80
2013	2,35
2014	2,54
2015	2,14
2016	1,84
2017	2,33
2018	2,91
2019	2,14
2020	0,89
2021	1,45
2022	2,95
Average	***2,12***

daily average %	***0,008***

- Sharpe Ratios

Portfolios	**Sharpe ratio**
1	0,01075
2	0,01358
3	0,01646
4	0,01938
5	0,02232
6	0,02526

7	0,02818
8	0,03107
9	0,03391
10	0,03665
11	0,03929
12	0,04180
13	0,04417
14	0,04611
15	0,04675
16	0,04734
17	0,04768
18	*0,04780*